



Lasers et optique non linéaire

# Lasers : Fundamentals



# Summary

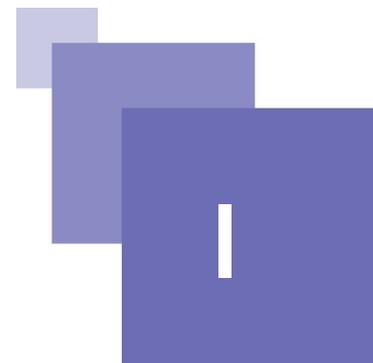


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# Lesson plan and main objectives



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This introductory lesson is intended for people who need to understand the basic principles of how lasers work and their main properties. It will also present the different types of lasers available today. The lesson plan is as follows: The introduction (Part I) describes the history behind the invention of the laser and how it was already well-established in the collective culture even before its discovery. Part II explains the basic mechanisms involved in the interaction between the laser medium and the light source. Part III focuses on the role of the optical cavity (or resonator) and how it influences the properties of the laser beam. Part IV details the different operating modes. In fact, Parts III and IV show how a laser beam is defined by specific spatial, spectral and temporal values. Part V describes the different types of laser, according to their amplifying medium. Finally, Part VI lists some of the possible applications.

## A. Introduction

The word laser is an acronym (an abbreviation pronounced as an ordinary word) of Light Amplification by Stimulated Emission of Radiation. Lasers are devices that produce or amplify a beam of narrow, low-divergence light with a well-defined wavelength within the optical region of the electromagnetic spectrum, covering the infrared, the visible and the ultraviolet. Historically, the laser is very different from other 20<sup>th</sup> century inventions as it first appeared as an imaginary literary creation.

### 1. A fantasy well-established in the collective culture

Even before the invention of the first laser in 1960, and when only a handful of specialists were aware of Einstein's prediction of stimulated emission (1917), the term “laser” was already well-known. In particular, numerous science-fiction writers had imagined an extremely powerful beam, capable of destroying anything in its path. These devices were generally controlled by aggressive intergalactic beings bent on wreaking havoc on the world.

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For example, in 1898, H.G. Wells describes in his book “The War of the Worlds” a “fiery ray” impressively similar to modern day lasers:

*«It was as if they were hit by an invisible ray that exploded into white fire on impact. Suddenly, it seemed they were all turned to flame, and I stood there stupefied, unable to comprehend that it was death that leaped from one man to another (...) I only had the impression that it was something very strange, this silent jet of light that knocked down everything it touched, and when this invisible line passed over them, even the pine trees and the bushes burst into flame with a deafening sound...»*

The “fiery ray” described by H.G. Wells is the incarnation of the absolute weapon: fired out of a vibrating funnel, a narrow ray straight as a rod sweeps over the surrounding countryside destroying everything in its path. It is hardly surprising that this fantasy of ultimate power (a “death ray” that quickly and “cleanly” kills the potential target from afar) was extremely popular during the first half of the 20<sup>th</sup> century and when the research scientist T.

Maiman announced that he had developed an actual prototype, there was a huge overreaction.

The public were carried away by the discovery: even the most serious scientific commentators waxed lyrical about the myth that had become a reality. M. Friedman (Science et Avenir) wrote:

*«When this shining light bursts forth from the centre of a stimulated ruby, the Academy of Sciences is just as surprised as the rest of us. It seems that austere physics has joined with seductive alchemy (...) In the heart of every crystal there sleeps a ray of light: this simple phrase is linked to so many myths and legends that people are burning with anticipation to see finally with their own eyes a beam that leaves blackened soil in its wake, the indispensable “death ray” that has made science fiction what it is today.»*

The demonstration of stimulated emission at the Hughes Laboratory in Malibu, California, was described enthusiastically by journalists: *«Suddenly a light from hell appeared in the middle of the ruby. Then, from the end of a cylinder, a hundred thousand times brighter than the sun, burst forth a thin red light, a perfectly parallel monochromatic beam (...) Theodore Maiman and his assistants were silent for some time, enthralled by the beauty of this spectacle, the like of which no-one had ever seen before. 'Einstein was right' he murmured, 'light can be concentrated and coherent'»*. In reality, Maiman had only observed a slight shrinking of the fluorescent spectrum, one of the effects of lasers, but much less spectacular for the layman.

## 2. Some important dates

Here are the important steps that led to the discovery of the laser:

- 1887: Heinrich Hertz accidentally discovers the photoelectric effect. This lucky breakthrough will allow Albert Einstein to introduce the notion of photons.
- 1901: The scientific conundrum known as “the ultraviolet catastrophe” (spectral energy densities diverge at high frequencies) is solved by Planck. He hypothesises that the energy of a type of frequency  $\nu$  is not a random continuous variable but a random discrete set of variables, represented by the values  $nh\nu$ . Interestingly, Planck and his contemporaries at first find it very difficult to accept this idea of discrete leaps in energy. However, subsequent experiments prove that the theory is entirely correct.
- 1905: Einstein introduces a means to quantify electromagnetic energy. The photon is born. Unfortunately, the arrival of the photon cannot take into account the phenomenon of black body radiation (the spectral density of electromagnetic energy emitted by an enclosed area at a temperature  $T$  and at thermal equilibrium). However, shortly afterwards, Born devises a means to quantify the energy levels of electrons (1913). This in turn allows Einstein to prove that photons and black body radiation are in fact compatible thanks to the notion of stimulated emission.
- 1949: Kastler and Brossel develop the first optical pumping and the first population inversion. By 1950, the first MASERS appear (Microwave Amplification by Stimulated Emission Radiation),

devices that are capable of amplifying an electromagnetic wave in the microwave region (Weber, Townes and Basov).

- 1954: The first MASER is built (an ammonia maser with a 13 mm wavelength). The electromagnetic wave is confined in three dimensions by a “box” and is reflected off its sides. However, this is still in the microwave rather than the optical domain. In fact, scientists at the time thought it was impossible to make an optical laser because the cavity would have to be incredibly small (of the order of magnitude of a wavelength i.e. only tens of  $\mu\text{m}$  at the most!).
- 1958: Schawlow and Townes decide to use an open Fabry-Pérot cavity for their experiments. The idea is to confine the electromagnetic field like in a closed box but in only one dimension: the main axis of light propagation in the cavity. This means that only certain specific electromagnetic waves are amplified, but the resulting beam is much more powerful than when using a closed cavity.
- 16<sup>th</sup> May 1960: Maiman demonstrates the first ever optical laser effect. The amplifying medium is a ruby, the crystal most used in early lasers because it was already well known from its application in MASERs. This is a pulsed operation laser with a wavelength of 694.3 nm.
- 1961: Javan, Bennet and Herriot build the first gas helium-neon laser operating continuously at 1.15  $\mu\text{m}$ . In fact, this laser can emit over a whole range of discrete wavelengths, from green to infrared via orange and red (633 nm).
- 1962: First red helium-neon laser.
- 1965: First semiconductor lasers.
- 1966: First coloured pulsed lasers (red, orange, yellow).
- 1970: First coloured continuous-wave lasers (red, orange, yellow).

Since the discovery of the first real laser in 1961, many others have been developed each year. Current research is focused on the development of solid state lasers (diode lasers, crystal or amorphous solids doped with active ions, optic fibre lasers) with the aim of obtaining much shorter pulses (the present limit is 4.5 fs or  $4.5 \cdot 10^{-15}$  seconds) and much greater power (emissions of about 10 kilowatts are now common).

### 3. Why invent the laser?

When it was first discovered, the laser gave rise to some foolish hopes and was heralded as the answer to all the world's problems: it could be used to melt dangerous icebergs, replace the existing telephone network or carry millions of volts from one place to another. Leaving aside the more far-fetched (or premature in the case of telecommunications) applications, by 1965 some people envisaged using lasers as scalpels in surgical operations, or to facilitate nuclear fission, or as a precision cutting tool for metals or as a means to store information (CDs) or to produce 3D images (holography).

Nevertheless, all these applications were initially impossible to put into practice so people began to wonder what was the use of this device. Strangely enough, contrary to most other 20<sup>th</sup> century inventions, the laser was not invented in response to an industrial, scientific or social need. For example, nuclear power was developed to meet the growing need for energy, computers were invented to carry out increasingly complex calculations, and quantum mechanics were formulated to solve some theoretical problems such as black body radiation or the photoelectric effect. The laser seemed to come out of nowhere, or almost. Of course, some famous theoreticians (Townes, Schawlow, Basov, Prokhorov and others) worked on the subject in order to transpose microwave amplifiers, called masers, into the optical domain. Other scientists were interested by Einstein's prediction of stimulated emission and tried to observe this effect experimentally. Nobody, however, actually “needed” the laser. Nobody was desperately waiting for it to be invented so they could put it to some use. The best example comes from Arthur Schawlow, one of the illustrious inventors of the laser: “One day all typewriters will be equipped with a laser to erase typing errors. The laser beam will vaporise the ink on the page in a fraction of a second without leaving the slightest trace.” So, the laser was destined to become just a “super-corrector

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for absent-minded typists”? In the early years following its invention, it was mocked by a good number of industrialists. Even high-ranking scientists got involved. Pierre Aigran (Secretary of State for Research, researcher and member of the Academy of Sciences at the time of the discovery of the laser) stated: “We are used to having a problem and looking for a solution. In the case of the laser, we already have the solution, we just have to find the problem.”

Of course, with the passage of time, numerous problems have been solved thanks to the laser and some of those early dreams have come true (telecommunications, laser weapons, fusion by laser, laser scalpels). What seemed to be just a cumbersome “toy” for researchers is today universally hailed as one of the most important inventions of the 20<sup>th</sup> century, both in terms of physics and its applications. The laser is now used in an ever widening range of sectors: treatment of materials, biomedical, instrumentation and measurement, laser shows.

### 4. What is a laser?

A laser consists of two fundamental elements:

- an amplifying or gain medium (this can be a solid, a liquid or a gas). This medium is composed of atoms, molecules, ions or electrons whose energy levels are used to increase the power of a light wave during its propagation. The physical principle involved is called stimulated emission.
- a system to excite the amplifying medium (also called a pumping system). This creates the conditions for light amplification by supplying the necessary energy. There are different kinds of pumping system: optical (the sun, flash lamps, continuous arc lamps or tungsten-filament lamps, diode or other lasers), electrical (gas discharge tubes, electric current in semi-conductors) or even chemical.

These two components are sufficient to amplify an existing light source. This is known as a laser amplifier. However, most lasers also incorporate an optical resonator (or cavity) in order to produce a very special radiation. Technically, the whole device is known as a laser oscillator, but this term is often shortened to simply “laser”. The laser oscillator uses reflecting mirrors to amplify the light source considerably by bouncing it back and forth within the cavity. It also has an output beam mirror that enables part of the light wave in the cavity to be removed and its radiation used.

The different components that make up a basic laser are illustrated in the diagram below (Figure 1).

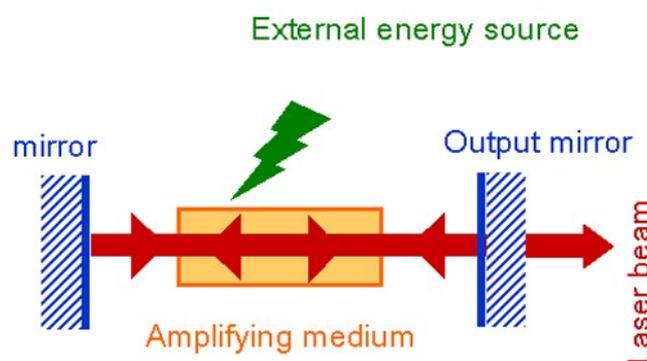


Figure 1: Diagram of a laser oscillator

## B. Basic mechanisms: emission, absorption and pumping

Stimulated emission is fundamental to light amplification and thus to the operation of the laser. To understand it, it must be placed in the context of interactions between light and matter. Here, the matter is composed of optically active elements in “solution” in a gas, plasma, solid or liquid medium. These elements can be atoms, ions, molecules, free radicals or electrons (for simplicity, we consider “atoms” in the following). Their energy levels are quantified and are such that light of a certain frequency can interact with the population found in these levels. More precisely, let us consider two energy levels  $E_1$  and  $E_2$  ( $E_1$  is less than  $E_2$ ) whose atoms can interact with light of frequency  $h\nu = E_2 - E_1$ . The group  $E_1$ - $E_2$  is called radiative transition if atoms can only pass from  $E_1$  to  $E_2$  (or from  $E_2$  to  $E_1$ ) by interacting with light.  $E_1$  is called the lower energy level and  $E_2$  the upper energy level.

### 1. The emission-absorption principle

The three different mechanisms are shown below (Figure 2):

1. Absorption: An atom in a lower level absorbs a photon of frequency  $h\nu$  and moves to an upper level.
2. Spontaneous emission: An atom in an upper level can decay spontaneously to the lower level and emit a photon of frequency  $h\nu$  if the transition between  $E_2$  and  $E_1$  is radiative. This photon has a random direction and phase.
3. Stimulated emission: An incident photon causes an upper level atom to decay, emitting a “stimulated” photon whose properties are identical to those of the incident photon. The term “stimulated” underlines the fact that this kind of radiation only occurs if an incident photon is present. The amplification arises due to the similarities between the incident and emitted photons.

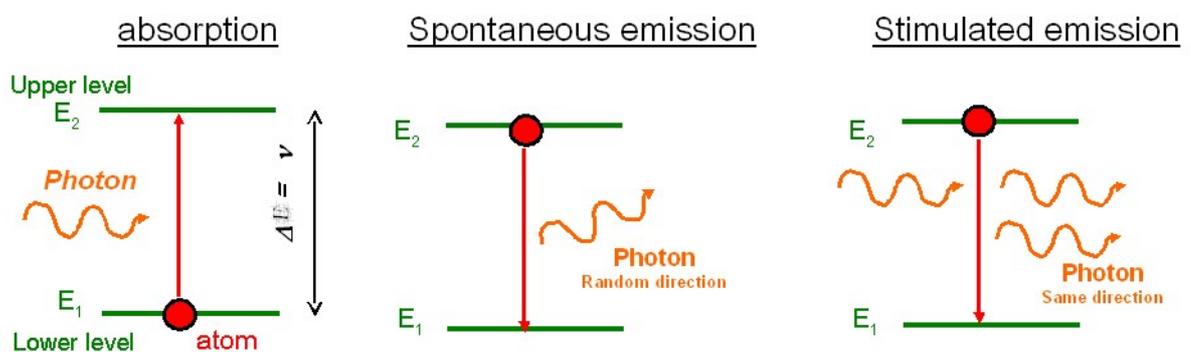


Figure 2: Mechanism of the interaction between an atom and a photon (The photon has an energy  $h\nu$  equal to the difference between the two atomic energy levels).

### 2. Competition between the three mechanisms

For a radiative transition, these three mechanisms are always present at the same time. To make a laser medium, conditions have to be found that favour stimulated emission over absorption and spontaneous emission. Thus, both the right medium and the right conditions must be chosen to produce the laser effect.

- An incident photon of energy  $h\nu$  has an equal chance of being absorbed by a ground-state atom

as being duplicated (or amplified!) by interacting with an excited-state atom. Absorption and stimulated emission are really two reciprocal processes subject to the same probability. To favour stimulated emission over absorption, there need to be more excited-state atoms than ground-state atoms.

- Spontaneous emission naturally tends to empty the upper level so this level has to be emptied faster by stimulated emission. It has been proved that stimulated emission is much more likely to happen if the medium used is flooded with light (i.e. with a large number of photons). A good way to do this is to confine the photons in an optical cavity.

### 3. Population inversion and pumping

If there are more atoms in the upper level ( $N_2$ ) than in the lower level ( $N_1$ ), the system is not at equilibrium. In fact, at thermodynamic equilibrium, the distribution of the atoms between the levels is given by Boltzmann's Law:

$$N_2 = N_1 \times \exp -((E_2 - E_1)/kT)$$

In this case,  $N_2$  is always less than  $N_1$ . A situation not at equilibrium must be created by adding energy via a process known as “pumping” in order to raise enough atoms to the upper level.



#### Définition

This is known as **population inversion** and is given by  $\Delta = N_2 - N_1$ . Light is amplified when the population inversion is positive. Pumping may be electrical, optical or chemical.

### 4. Spectroscopic systems used to create a laser

Not all atoms, ions and molecules, with their different energy levels, are capable of creating a population inversion and a laser effect. Only radiative transitions (where the atoms are excited due to light absorption) should be used and non-radiative transitions should be avoided. Some transitions have both a radiative and a non-radiative part. In this case, the upper level empties as a result of a non-radiative effect as well as spontaneous emission. This leads to additional problems for achieving a population inversion because it is difficult to store atoms in the upper level under these conditions. Thus, this type of transition should also be avoided.

Next, the relative energy levels specific to each type of atom must be considered. For example, choosing a lower level with more energy than the ground state will greatly limit the population  $N_1$ , which may even be zero (Figure 3). This means that only one atom would have to be excited to achieve population inversion.

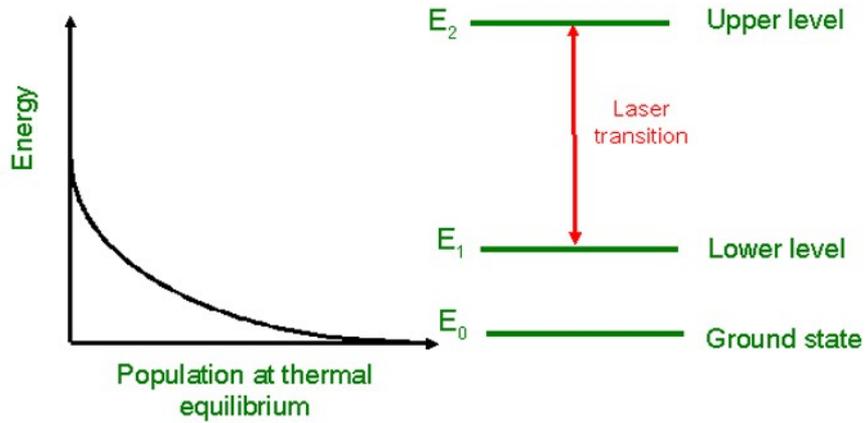


Figure 3: Laser transition with the lower level far above the ground state. The population at thermodynamic equilibrium is defined by Boltzmann's Law.

In addition, pumping must be able to move atoms to a higher level. Every pumping system (particularly optical or electrical) corresponds to a certain energy, which must be transferable to the atoms of the medium. The difference in energy between the excited state and the ground state must match the pumping energy. In optical pumping, there must be at least three different energy levels to create a population inversion. Figure 4 illustrates such a system. It shows the pumping transition (between  $E_1$  and  $E_3$ ) and the laser transition (between  $E_2$  and  $E_1$ ). The objective is to store atoms in level  $E_2$  by absorbing “pumping” radiation whose wavelength is shorter than that of the laser transition. This means that the excited atoms must quickly decay from level 3 to level 2 only, a condition that limits the choice of systems that will work. Figure 4 also shows an ideal cycle for an atom: it rises into level 3 by absorbing a photon from the pumping light. It then falls very rapidly into level 2. Finally, it decays by stimulated emission to level 1. Despite its simplicity, this is not a very easy system to implement as the ground state of the laser transition has a large population at thermodynamic equilibrium and at least half of this population must be excited to level 2 to obtain population inversion. Moreover, level 2 must be able to store these atoms so spontaneous emission must be very unlikely. This affects the choice of the system. A large pumping energy is also needed. The first ever laser was of this type and used a ruby ( $\text{Cr}^{3+}:\text{Al}_2\text{O}_3$ ). Ruby is composed of an aluminium crystal matrix and a doped ion ( $\text{Cr}^{3+}$ ) whose energy levels are used to create the laser effect. The medium is strongly pumped by discharge lamps.

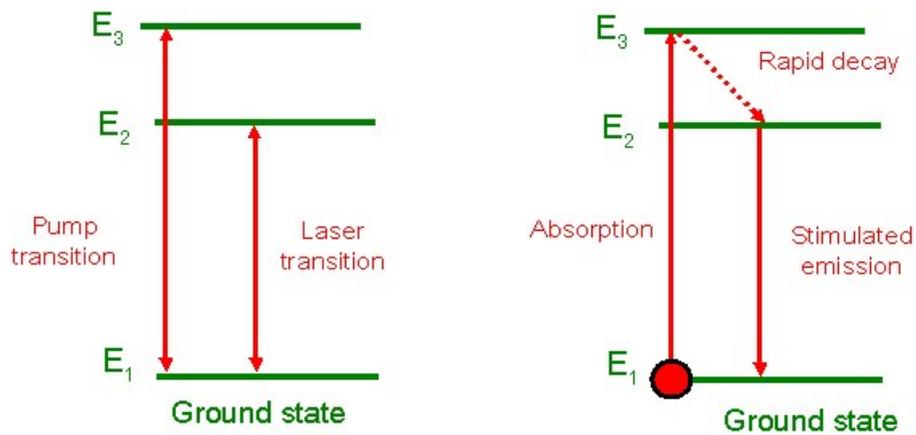


Figure 4: Example of a three-level system with optical pumping.

Another example of a spectroscopic system is the four-level laser (Figure 5). Here, the pumping transition (optical pumping) and the laser transition occur over a pair of distinct levels ( $E_0$  to  $E_3$  for the pump and  $E_1$  to  $E_2$  for the laser).  $E_1$  is chosen to be sufficiently far from the ground state  $E_0$  so that the thermal

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population at thermodynamic equilibrium is negligible. Similarly, atoms do not stay in level 3 or level 1. Figure 5 represents an ideal four-level system. Unlike the three-level system, as soon as one atom moves to level 2, a population inversion occurs and the medium becomes amplifying. To maintain the population inversion, atoms must not accumulate in level 1 but must rapidly decay to level 0. One of the best known mediums operating in this way is neodymium YAG ( $\text{Nd}^{3+}:\text{Y}_3\text{Al}_5\text{O}_{12}$ ).

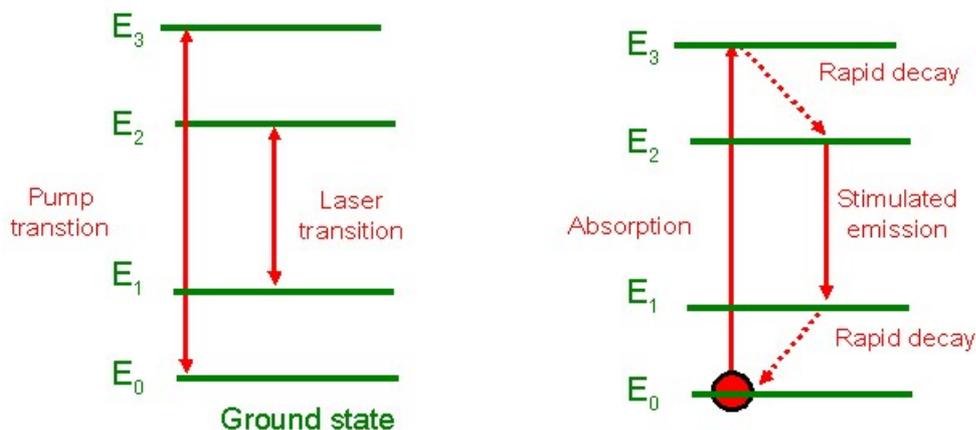
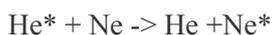


Figure 5: Example of a four-level system with optical pumping.

A final example of a spectroscopic system providing a laser effect is the helium-neon gas system (Figure 6). In this case the pumping method is electrical. Neon transitions are used for the laser transitions: there are several but the most well-known is the coloured one at 632.8 nm. Helium is used as an intermediary gas, capable of transferring energy from the electrons to the neon particles via collisions. Helium is also unique in having two excited states said to be “metastable” i.e. atoms can stay there a long time before falling to the ground state. Helium atoms are carried into the excited state by collisions with electrons. Energy is easily transferred to neon when the atoms collide because these metastable levels coincide with the excited states of neon. This process is given by the equation:



An excited helium atom meets a ground-state neon atom and transfers its energy while decaying.

Figure 6 also shows that the lower levels of the laser transitions are far from the ground state, which favours population inversion (no thermal population).

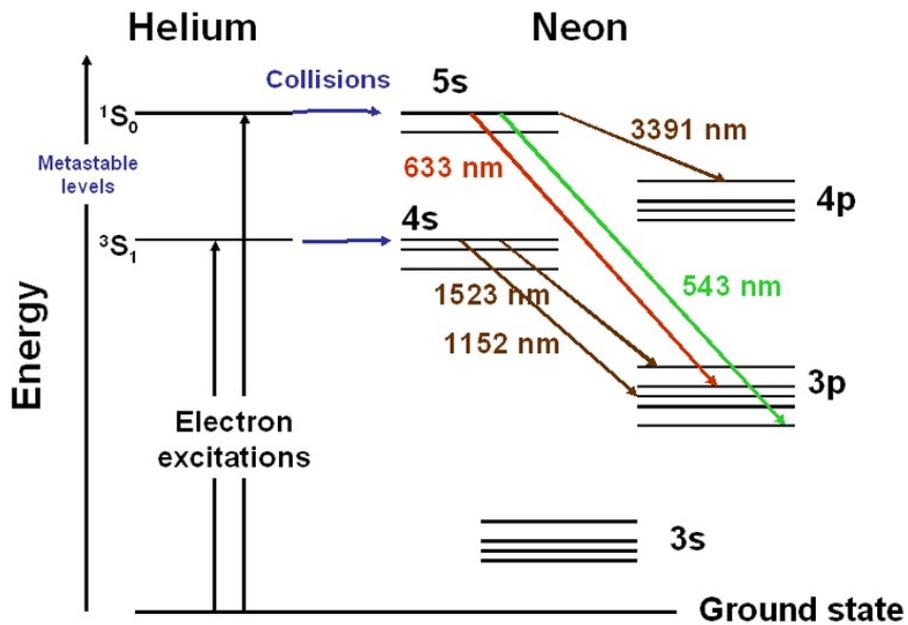


Figure 6: A Helium-Neon System

### C. The role of the optical cavity

The previous section showed how to favour population inversion by choosing the right spectroscopic system and energy levels. However, population inversion is not enough to generate a laser effect. As stated previously, stimulated and spontaneous emissions are competing with each other. Thus, before becoming an amplifying medium, a laser medium pumped by an external energy source is first a “lamp” (spontaneous emission). It is the optical cavity that creates the conditions necessary for stimulated emission to become predominant over spontaneous emission. The cavity or resonator is composed of several mirrors that bounce the beam back and forth through the amplifying medium. There are two different types (Figure 7): linear cavities (light is reflected back and forth) and ring cavities (light circulates round and round). The first type will be studied here.

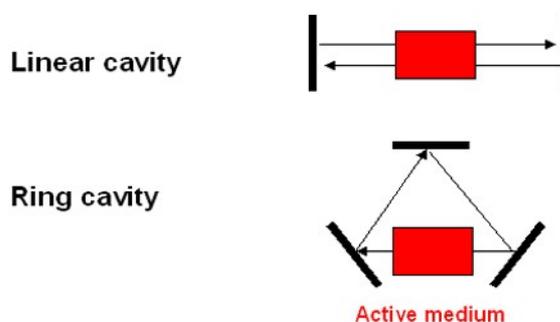


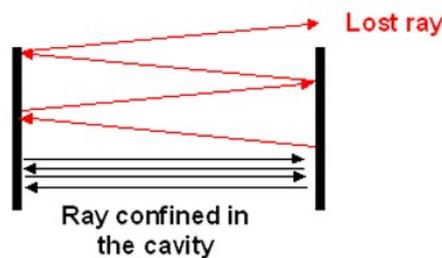
Figure 7: The two types of optical cavity.

When the laser starts up, the “lamp-amplifying medium” emits spontaneously in all directions. However, a small part of the emission occurs along the axis of the laser cavity. These spontaneous photons can

travel backwards and forwards. Thus, over time, thanks to the amplifying medium, the amount of light in the cavity increases considerably. The confinement of the light increases the probability of stimulated emission rather than spontaneous emission occurring. At the same time, the cavity acts as a filter due to the numerous round trips: only the wave perfectly perpendicular to the axis of the cavity will be propagated and certain frequencies will be favoured (the resonance frequencies of the cavity). In this way, the cavity produces a specific radiation.

### 1. Spatial characteristics of the emitted laser beam

The cavity acts as a spatial filter by selecting only those light rays beams close to its central axis: the others are lost due to their distance from the axis and the size of the mirrors (Figure 8).



*Figure 8: Behaviour of a non-perpendicular light beam in an optical cavity*

A laser operating in a steady state produces a light wave whose spatial structure does not change despite numerous round trips inside the cavity. In this case, the laser cavity must contain a light wave able to propagate in the cavity and remain constant after each round trip. This is known as a “Gaussian” wave whose light distribution is Gaussian in shape in the plane perpendicular to the axis of propagation. Physically, a Gaussian wave concentrates the light along the axis of the cavity. A Gaussian wave emitted through space is like a narrow beam of light and is called a Gaussian beam. By placing a small piece of cardboard or a detector perpendicular to the propagation axis of the wave (at the laser output) it is possible to measure the irradiance (the number of photons incident on a surface per unit area). Graphically, this irradiance will follow a Gaussian curve (Figure 9).

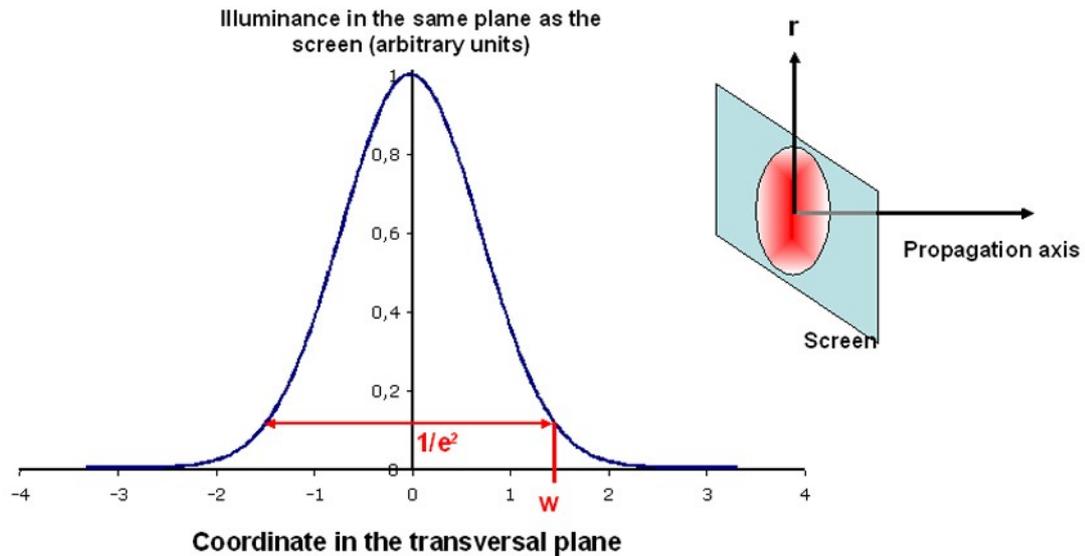


Figure 9: Appearance of a Gaussian beam: distribution of illuminance in a plane perpendicular to the direction of propagation.

A certain spatial extension of the light wave can also be defined: the radius of the beam ( $w$ ) is equal to the distance between the optical axis and the spot where the irradiance is divided by  $1/e^2$  in relation to the maximum irradiance of the wave.

A Gaussian wave propagates in a slightly different way from that described by classic geometrical optics. It has a minimum  $w_0$  at one place along the beam axis, known as the beam waist (Figure 10). Far from the waist, the beam diverges “in a straight line” at an angle of divergence  $\theta$ . The divergence and the radius are correlated by the formula:

$$\theta = \lambda / (\pi \times w_0)$$

A helium-neon laser, for example, has a radius of approximately 1 mm at the beam waist, which corresponds to a very low divergence of 0.2 mrad (the beam must travel 5 m from the waist for its radius to double!). This is, of course, impossible for beams of light emitted by ordinary lamps.

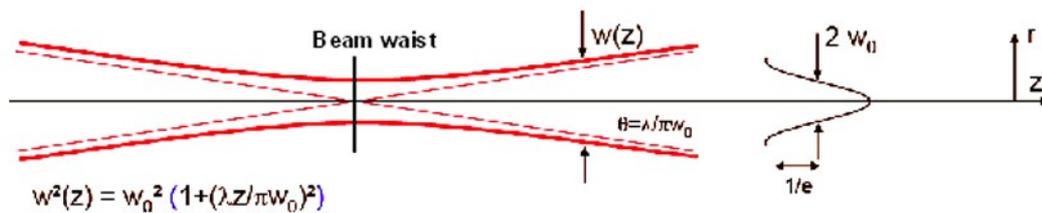


Figure 10: Appearance of the light beam according to its position ( $z$  is the propagation axis).

The above formula also proves that if the divergence is high (for example if the beam is focused by a lens) then the radius of the beam at the waist is very small. Generally, it is possible to focus the laser beam at a radius of the same magnitude as the wavelength. This can also be done with an ordinary lamp but the difference is the number of photons that can be delivered per second onto a small area. This is very low for an ordinary lamp but huge for a laser. For example, a 633 nm beam with a power of 1 mW corresponds to a flux of  $10^{15}$  photons per second and can easily be focused on a micrometre-wide spot (Figure 11). Thus, the power density of a simple helium-neon laser at a focal point is much greater than a sunbeam focused by a lens.



### Exemple

A striking example is the irradiance of a helium-neon laser emitting at 1 mW on the retina (equal to more

than  $100 \text{ W/cm}^2$ ) whereas the irradiance of a focused sunbeam is only  $10 \text{ W/cm}^2$ .

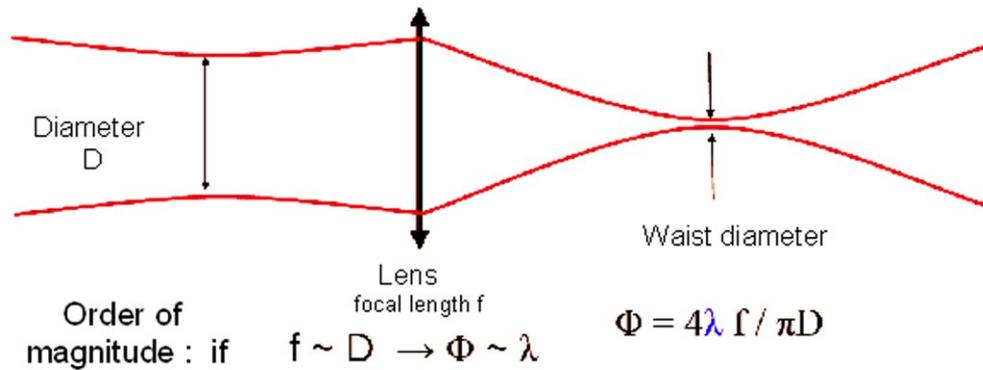


Figure 11: Order of magnitude of a Gaussian beam focused by a lens

### Conclusion

To summarise, an optical cavity selects a specific beam (a Gaussian beam) from the many photons spontaneously emitted by the “lamp-amplifying medium” and the number of photons carried by this beam is increased considerably, as it travels back and forth, by the process of stimulated emission. This beam can have a very low divergence and can be very precisely focused if the right optical tools are used.

## 2. The spectrum of a laser oscillator

As previously stated, the cavity also filters the spectrum emitted by the laser. A linear cavity is basically the same as a Fabry-Pérot interferometer. Only waves of a certain frequency can be successfully propagated. This frequency is defined by  $\nu = kc/2L$  where  $k$  is an integer,  $c$  the speed of light in a vacuum and  $L$  the optical length of the (linear) cavity. In the case of optical frequencies,  $k$  is very large and may reach tens of thousands for a cavity of a few centimetres. Waves that propagate with these frequencies in the cavity are known as longitudinal modes.



### Remarque

In the case of a ring cavity, the frequency is defined by  $\nu = kc/L$  where  $L$  is the optical length of the cavity circumference.

This filter is directly applied to the spectrum spontaneously emitted when the laser starts up. Progressively, the frequencies that cannot exist in the cavity disappear leaving only those that verify the equation above.

The spectrum emitted by a laser oscillator is thus composed of a comb of regularly spaced  $(C/2L)$  frequencies, usually centred on the spontaneous emission spectrum (Figure 12).

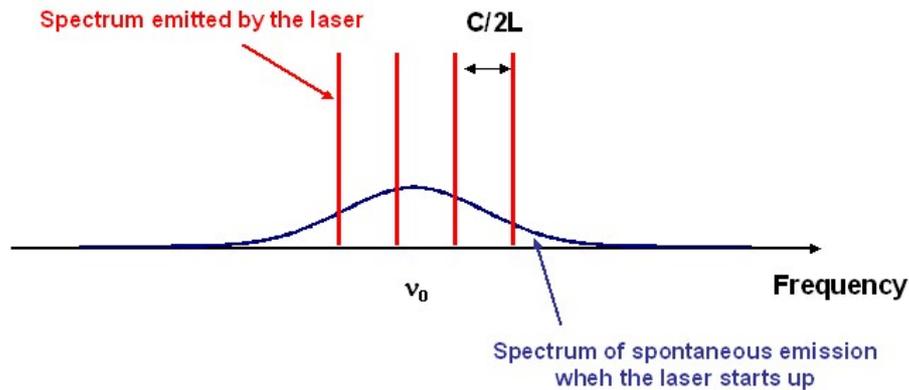


Figure 12: Appearance of the emitted spectrum of a laser compared to the spontaneous emission of a laser transition.

A laser is often described as monochromatic (for example, the helium-neon laser), a definition that must be well understood. In fact, broadly speaking, the spectral bandwidth of a laser is given by the width of the spontaneous emission: if the transition between the upper and lower levels is narrow, then the spontaneous emission will be fractions of a nanometre (this is the case for the red line in neon, which has a width equal to  $1/1000^{\text{th}}$  of a nanometre and a frequency of 1 GHz). The spectrum of a helium-neon laser is therefore “monochromatic” in the sense that only one colour is visible to the naked eye as the line is very narrow. Other types of laser have a much wider transition (for example, several hundreds of nanometres for the titanium-doped sapphire, which has a spontaneous emission spectrum ranging from 700 to more than 1000 nm) and consequently emit a spectrum that cannot be defined as monochromatic.

The spectral properties of lasers become even more interesting when just one frequency can be selected (using a series of filters placed in the optical cavity). This type of laser is defined as a single frequency or single mode laser. In this case, the width of the spectrum can be very much smaller than the spontaneous emission spectrum. For example, some helium-neon lasers have a spectral width of 1 Hz while the linewidth is measured in GHz.

To summarise, the optical cavity is capable of filtering the spontaneous emission in the form of discrete frequencies (the longitudinal modes). If only one mode is selected, the resulting laser radiation is of very high quality: a large number of photons are emitted in a very narrow beam only several Hz in width!

### 3. Operating conditions for the cavity

#### a) Introduction

The two previous sections have shown that a laser beam can be considered as a “concentrated spatial and spectral beam of light” and that the optical cavity plays a very important role. However, there are several conditions that must be fulfilled if the laser is to operate successfully. These involve the gain and loss of the cavity and the frequency.

#### b) Conditions for the gain medium

The effective gain of an amplifying medium is defined by the ratio between the power of the outgoing beam  $P_{\text{out}}$  and the incoming beam  $P_{\text{in}}$ :

**Lesson plan and main objectives**

$$G = P_{out} / P_{in}$$

These two powers (expressed in watts or photons per second) refer to the beam before and after it passes through the amplifying medium (Figure 13).

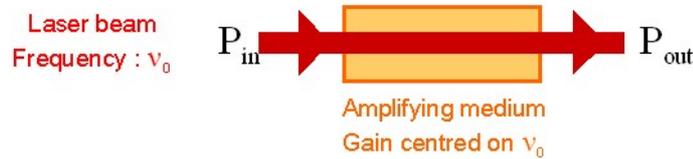


Figure 13: Power of the laser before and after passing through the amplifying medium

Reflection coefficients can also be defined for the mirrors in the cavity ( $R_1$  and  $R_2$ ), supposing that the only loss of power is due to the reflections off these mirrors (Figure 14).

When the laser is operating continuously, the output power is constant despite the fact that the number of photons in the cavity increases when passing through the amplifying medium then decreases when reflected off the mirrors. Thus, the number of photons gained is equal to the number of photons lost.

$P$  is the power of the laser just before the mirror  $M_1$ . The power after a round trip is given by:

$$P_{RT} = G_+ R_2 G_- R_1 P$$

where  $G_+$  and  $G_-$  are the effective gains in the positive (outgoing laser beam) and negative (incoming) directions, respectively.

It is necessary to distinguish the effective gains according to the direction of the wave propagation as this depends on the incident power, which is not the same in both directions (the reflection coefficients of the mirrors are different).

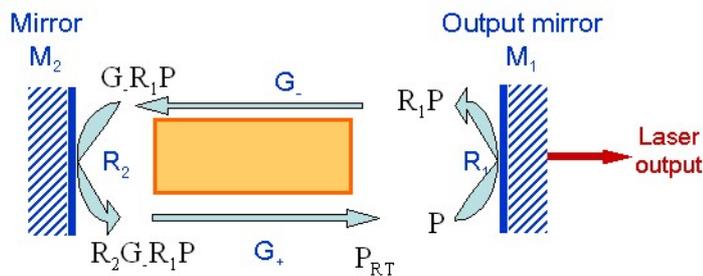


Figure 14: The amplifying medium in the optical cavity: the effect of mirrors and the gain on the laser beam.

When the laser emits continuously,  $P = P_{RT}$  so the product

$$G + G - (\text{the gain after a round trip}) = 1 / (R_1 R_2) :$$

If  $G + G - < 1 / R_1 R_2$ , the laser cannot oscillate.

If  $G + G - > 1 / R_1 R_2$ , the power of the beam in the cavity increases each time. This increase is not infinite as the number of excited-state atoms is given by the pumping so the number of stimulated

photons is finite. Thus, if the maximum number of photons that can be pumped per second is  $N$ , the effective gain can be defined by the formula  $G = (P_{in} + N) / P_{in}$  where  $P_{in}$  is the power of the beam just before the amplifying medium (in photons per second). If  $P_{in}$  increases, the effective gain decreases and tends towards 1. This phenomenon is known as gain saturation. Thus, when the power in the cavity increases significantly,  $G + G^-$  decreases and finally stabilises at  $G + G^- = 1 / (R_1 R_2)$ .

The cavity mirrors must therefore be chosen so that  $G + G^-$  is always more than 1 or, in other words, so that the gain is always greater than the loss in the cavity (represented by the transmissions of the mirrors).

### c) Conditions for the optical frequency

The various frequencies that can exist in the cavity are defined by the formula  $\nu = kc/2L$ . They must also be situated in the bandwidth of the amplifying medium. In fact, the product  $G + G^-$  has a certain bandwidth depending on the nature of the medium (for example, the helium-neon laser has a bandwidth of about 1 GHz). This means there is a certain spectral range defined by  $\Delta\nu$  where the frequencies will oscillate. These conditions of gain and frequency can be correlated graphically (Figure 15).

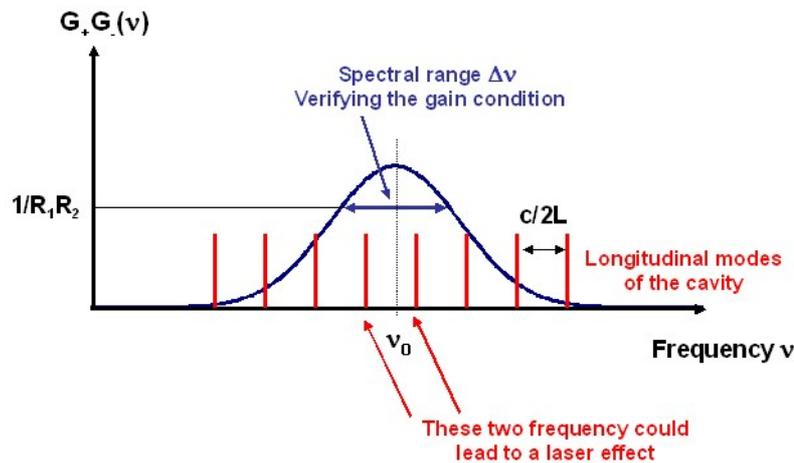


Figure 15: Conditions for the gain and the frequency (in the case shown here, two frequencies may oscillate in the laser).

In the frequency comb imposed by the cavity, only those frequencies that meet the above conditions can lead to a laser oscillation. The others cannot exist. Generally,  $\Delta\nu$  is greater than  $c/2L$ . This means that quite a large number of frequencies will be able to oscillate (tens to hundreds of frequencies). The actual number depends on the bandwidth defined by  $\Delta\nu$  and the spectrum imposed by the Fabry-Pérot cavity ( $c/2L$ ).

However, in some cases,  $\Delta\nu$  is smaller than  $c/2L$  (Figure 16). This often happens when the amplifying medium emits in a very narrow band (in  $\text{CO}_2$  lasers for example) or when the optical cavity is very small (microchip lasers have a cavity smaller than 1 mm). As a result, no frequency may be able to oscillate so the length of the cavity must be adjusted so that the laser can work.

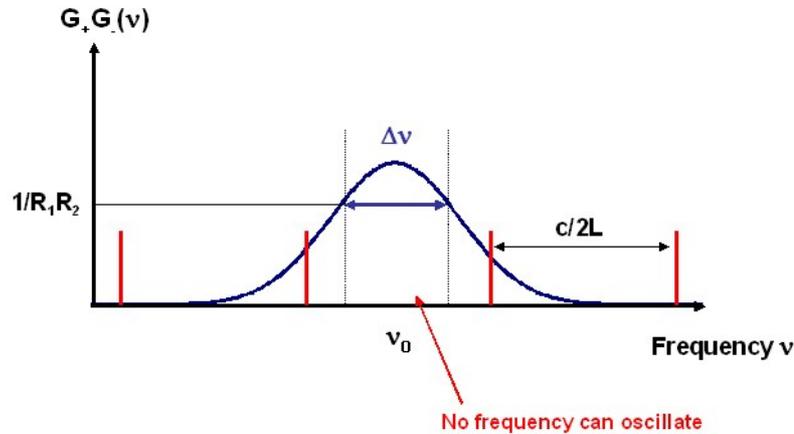


Figure 16: An extreme case where the spectral range  $\Delta \nu$  is smaller than  $c/2L$

## D. The different temporal operating conditions

The previous section shows that laser radiation is a concentrated spatial and spectral beam of light. It is also possible to qualify the beam as a “temporal concentration” by considering that the emitted photons are condensed into short energetic pulses. As previously stated, the probability of obtaining stimulated emission increases with the number of excited-state atoms and incident photons. There are therefore two ways to favour stimulated emission: either by raising the number of excited-state atoms or by raising the number of incident photons. The first method to achieve a temporal concentration is to trigger the stimulated emission only when there is a large number of atoms in the upper level (Q-switching). In the second method, the photons in the optical cavity are condensed into a “packet” or “pulse” that will bounce back and forth between the mirrors (mode-locking).

### 1. Q-switching

In order to store many atoms in an upper level, the flow to a lower level must first be limited. Thus, stimulated emission must be prevented by placing an attenuator in the cavity to stop light from travelling back and forth (note: this attenuator is usually a light modulator, rather than a mechanical shutter, which reduces the amplitude or power of the light beam). In this case, for a radiative transition, the only decay to a lower level is due to spontaneous emission. When the pumping system supplies more atoms per second than lose energy by spontaneous emission, the population in the upper level can become very large (Figure 17).

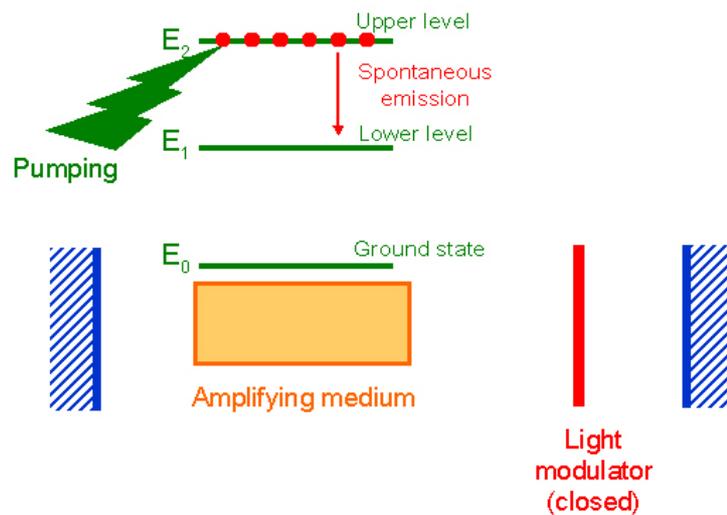


Figure 17: Accumulation of atoms in the upper level when the optical cavity is blocked.



### Remarque

This operating condition is much easier to achieve with media that have a low rate of spontaneous emission. This is true for solid state ion-doped lasers (for example Nd:YAG or Yb:YAG) but not for gas (neon or argon) or semiconductor lasers. These have high rates of spontaneous emission so it is difficult to attain a large population in the upper level.

After a certain time, the energy losses in the cavity are suddenly reduced so that laser oscillation becomes possible. As there is a very large population in the upper level, stimulated emission becomes very probable and the laser is suddenly triggered. The flow due to stimulated emission is much greater than the other flows (filling by pumping and emptying by spontaneous emission): all the atoms stored in the upper level fall sharply, emitting stimulated photons (starting with the spontaneous emission trapped in the cavity). Thus, the laser cavity fills with stimulated photons at the same time as the upper level empties (Figure 18).

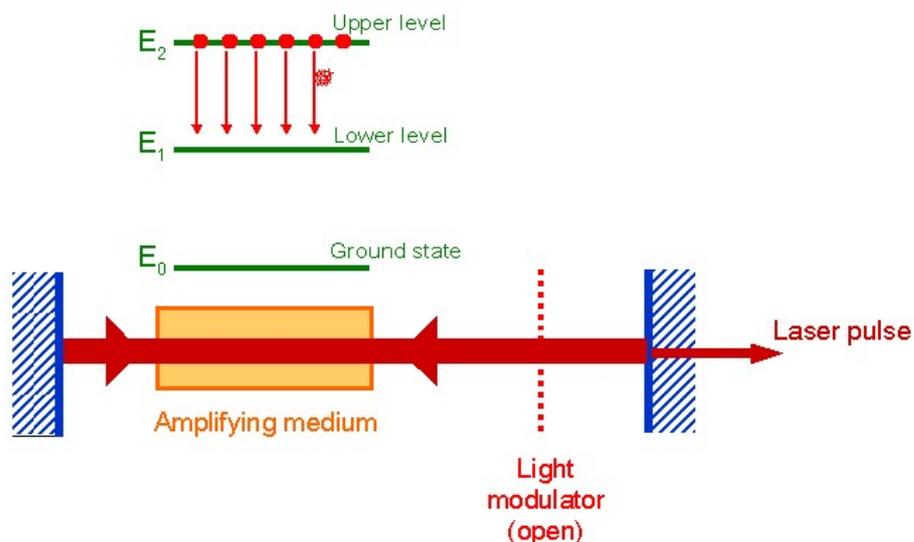


Figure 18: Laser effect once the optical cavity is suddenly opened.

## Lesson plan and main objectives

Eventually, the upper level is completely empty. There is no further stimulated emission and the cavity will also empty due to the losses created by the output mirror (in general, the cavity empties after only a few round trips)(Figure 19).

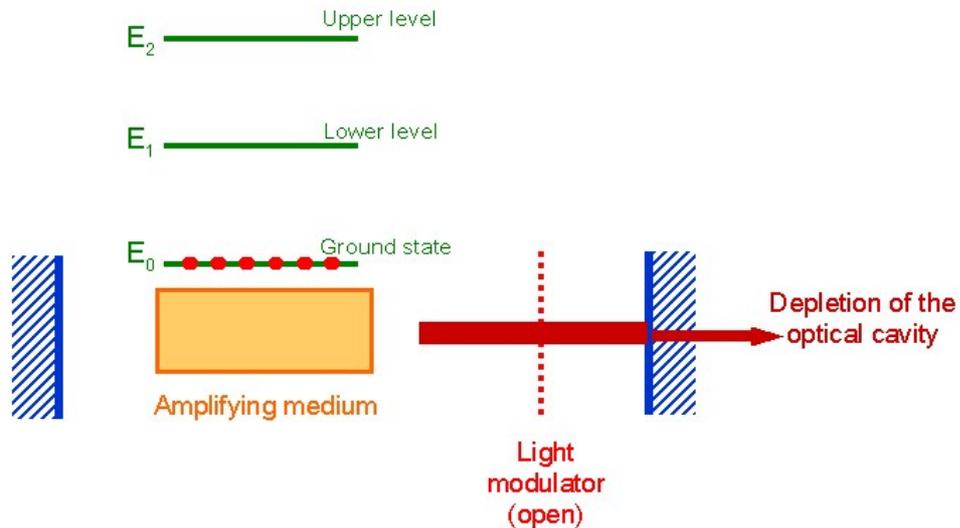


Figure 19: Depletion of the optical cavity once all the atoms have returned to the ground state.

This process gives rise to a dramatic variation in the number of photons in the cavity (first by a significant amplification due to stimulated emission then by the complete emptying of the cavity at the end). The net result is the emission of a short pulse of light via the output mirror.

Generally, several round trips are needed to completely depopulate the upper energy level and several more round trips to empty the optical cavity so the duration of the pulse is greater than one round trip. This means that for optical cavities shorter than a metre (one round trip less than 6 ns), it is possible to generate short pulses of only a few nanoseconds but several millijoules in power. The peak power (the pulse energy divided by its duration) of these lasers can be in the megawatt range or even higher.

It should be noted that Q-switched lasers never reach a steady state as they stop functioning after several round trips of the light in the cavity.

## 2. Mode-locking

The second operating technique is completely different. This time, the laser oscillator is left to reach a steady state and the oscillation in the cavity is not blocked. However, the cavity is prevented from filling with photons everywhere at the same time: only a packet of photons is allowed to propagate in the cavity. This pulse lasts for a shorter time than a round trip in the cavity. In other words, its spatial extension is markedly shorter than the length of the cavity.

The method used to obtain these operating conditions consists in using a rapid light modulator that can chop the light in the cavity into periods of exactly the same length as a round trip. Thus, only those photons allowed to pass through the modulator in its on-state will be amplified and will always find the modulator in this state after each round trip. The other photons elsewhere in the cavity will be subject to losses when they travel through the modulator (Figure 20).

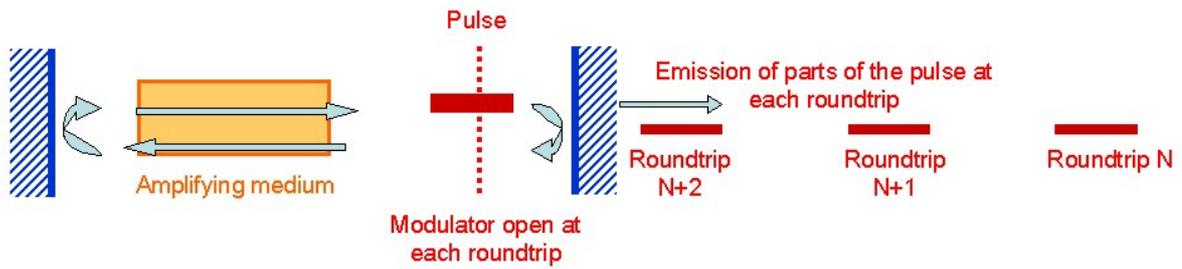


Figure 20: A pulse propagating in the optical cavity of a mode-locked laser.

Generally, the pulses last for a much shorter time than a round trip in the cavity. They are limited by the Fourier transform of the spectrum emitted by the laser: the wider the spectrum, the shorter the pulse. This means that if the amplifying medium is exceptionally wide (for example the titanium-doped sapphire has a spectral width greater than 300 nm), then the pulse generated will be only several femtoseconds long.

Figure 20 shows only a single pulse travelling in the cavity. However, a pulse train can be seen leaving the laser, generated each time the pulse hits the output mirror. The pulse repetition period corresponds to the cavity round-trip time (typically several nanoseconds).

The average power of a mode-locked laser is of the same order of magnitude as that of continuous-wave lasers. In fact, in contrast to Q-switched lasers, these can also reach a steady state like continuous-wave lasers. The fundamental difference is that the stimulated photons are condensed in a packet rather than spread all around the cavity. During one round trip, only one laser pulse is emitted via the output mirror. The pulse energy is thus equal to the average power multiplied by the duration of a round trip. Generally, these energies are of the order of several nanojoules.

The term “mode-locking” comes from the analysis of the various frequencies. A laser operating under these conditions will emit over several different frequencies due to the rapid modulation of the modulator. They are also imposed by the optical cavity, spaced out by  $c/2L$ : the longitudinal modes of the cavity.

In fact, contrary to what common sense would predict, the longitudinal modes interfere even if they have different frequencies because they co-exist spatially. For example, if the laser emits continuously at two frequencies separated by  $c/2L$ , the light output due to interference of the two waves will be modulated by a sinusoidal term of frequency  $c/2L$ . This modulation is generally very rapid (a few nanoseconds for a metre-long cavity). It can only be detected by sufficiently sensitive equipment (a fast photodiode and a rapid oscillator). The phenomenon is known as beating and results from the interference of beams with different frequencies.

When a large number of frequencies are emitted by the laser, the beat signal becomes quite complex. Its shape depends on the relative phase of the waves with different frequencies. However, the beat signal has a very regular shape in one particular case: when all the waves emitted by the cavity are in phase. Then, there are certain times and spots in the cavity where all the waves beat in phase and the interference signal is thus very powerful (Figure 21).

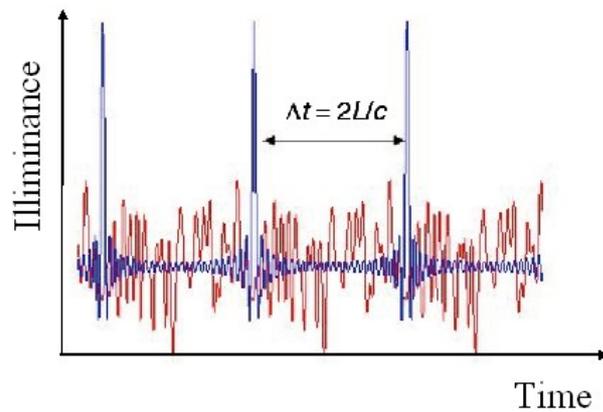


Figure 21: Comparison of the beat signal (on leaving the cavity) when all the modes are in phase (in blue) and with random phases between the modes (in red).

When the longitudinal modes are in phase, there is only one place in the cavity where the electric fields add together constructively. Everything occurs as if a pulse was travelling inside the cavity, just as described at the beginning of this section (Figure 22).

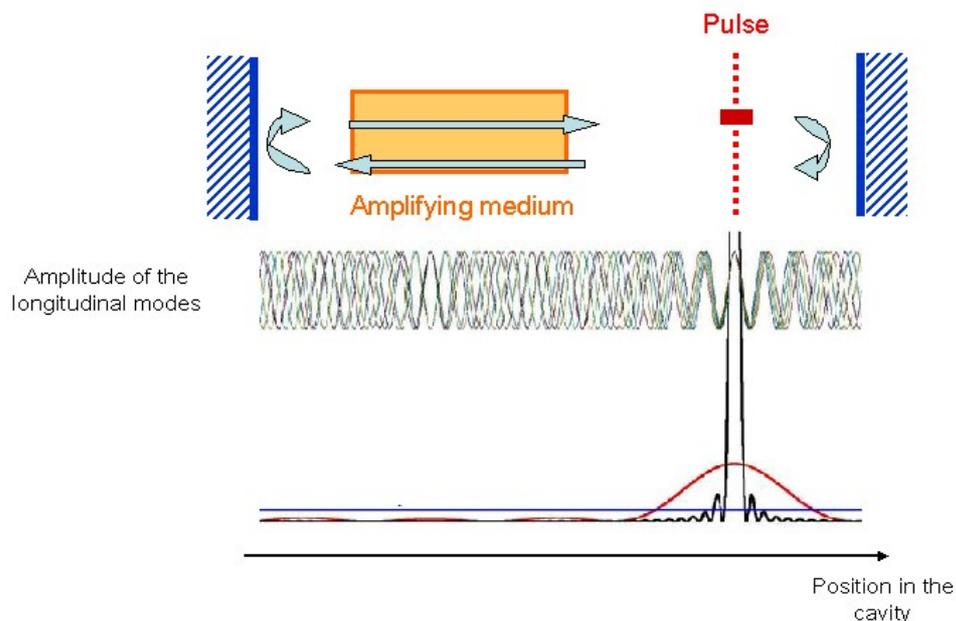


Figure 22: "Snapshot" at a given moment. The different sinusoidal curves represent the amplitude of the electrical field for different modes of the cavity.

### 3. Other operating modes (pulsed pumping)

There are other operating modes that are based on changes made to the pumping system rather than to the cavity. Naturally, when the pumping is pulsed, the laser emission is also pulsed. When there is only the amplifying medium in the cavity (no modulator), the operating mode is defined as gain-switching. Generally, pulses that occur during gain-switching are longer and weaker than Q-switched lasers.

In the case of diode lasers, pumping can easily be modulated by pulses of electric current. The gain in the

semiconductor is directly correlated to the different modulations and the process is known as gain switching. The pulses can be very short (only several picoseconds) but the energy released is very low (of the order of only  $10^{-12}$  J).

## E. Different types of laser

The different types of laser can be classified according to the nature of the amplifying medium: gas, liquid(dye) or solid state. The aim of this section is to give an overview of the principal families of lasers in terms of their uses.

### 1. Gas lasers

Gas lasers all have in common the same pump source: electricity. The gaseous species enter the excited state either directly, by collision with electrons, or indirectly, by collision with other gases, themselves electrically excited.

Gas lasers cover the whole optical spectrum, from the ultraviolet to the far infrared. However, the spectrum is not continuously covered: gas lasers emit very narrow spectral lines. The most common gas lasers (from the UV to the far IR) include:

- excimer lasers (ArF:193 nm, KrF:249 nm, XeCl:308 nm)
- argon-ion lasers (blue and green wavelengths)
- helium-neon lasers (the neon is used for the laser effect) 632.8 nm, 543.3 nm, 1.15  $\mu\text{m}$ , 3.39  $\mu\text{m}$
- CO<sub>2</sub> lasers: a large number of wavelengths around 9.6  $\mu\text{m}$  and 10.6  $\mu\text{m}$ .

Only CO<sub>2</sub> lasers are really efficient (15 to 20%). They are used in industry for processing materials. The efficiency of the others is mostly less than 1%. Gas lasers are often bulky and need a great deal of water-cooling (almost all the energy provided by the pump is lost as heat). Even though those operating in the visible (Argon, Helium, Neon) are tending to be replaced by solid state lasers, excimer lasers and CO<sub>2</sub> lasers are still very frequently used (for the treatment of materials in the broadest sense).

### 2. Dye lasers

Dye lasers use organic materials that generally emit in the visible spectrum and are thus coloured. These molecules are diluted in a solvent (usually an alcohol, like ethylene glycol or methanol).

The pump source of dye lasers is optical: either an arc lamp or, in the majority of cases, another laser (gas or solid state).

The whole of the visible spectrum is covered. In fact, the dyes are complex organic molecules that have many energy levels. The levels are so close together that they are considered as an energy band. In general, a molecule of dye covers continuously a region of about fifty nanometres in the visible. Dye lasers are the only ones to cover the visible spectrum entirely. Despite these interesting properties, dye lasers are little used because their implementation is impractical: to prevent the molecules from being destroyed by the pump source, the dye circulates in the pumping zone from a reservoir. In addition, the dye and solvent mixture degrades with time and must be changed regularly.

### 3. Solid state lasers

#### a) Introduction

Solid state lasers are either semiconductor (or diode) lasers pumped electrically or those with a crystalline or glass matrix pumped optically.

#### b) Diode lasers

Diode lasers use the recombinations between the “electron-hole” pairs found in the semiconductors to emit light in the form of stimulated emission. The pump source is electrical with an efficiency that can reach 60%. The wavelength can cover from the near UV to the near infrared depending on the materials chosen (GaN, GaAlInP, AlGaAs).

These are the most compact (the cavity uses the cleaved sides of the semiconductor and is barely 1 mm long) and the most efficient lasers available. The power can now reach several kilowatts by putting together hundreds of diode lasers and combining them in the same optic fibre. The only disadvantages of these diode lasers are the poor spatial quality of the emitted beam and that they cannot operate at a pulsed rate (Q-switching, see section IV).

#### c) Other solid state lasers

Other solid state lasers can compensate for the disadvantages of diode lasers. They use matrices that cannot conduct current so cannot be pumped electrically. They are pumped optically by either diode lasers or arc lamps (flash lamps). The matrices are doped with ions whose transitions provide the laser effect ( $\text{Nd}^{3+}$ ,  $\text{Yb}^{3+}$ ,  $\text{Er}^{3+}$ ,  $\text{Ti}^{3+}$ ). In general, solid state lasers emit in the red and near infrared. Of particular interest is the wavelength of  $\text{Nd}^{3+}$ :YAG( $\text{Y}_3\text{Al}_5\text{O}_{12}$ ) with an emission at 1064 nm.

Following the host and the ions used, the emission spectra can be narrow (fraction of nm) or wide (hundredth of nm).  $\text{Tr}^{3+}$  : sapphire is one of the material having the largest spectrum : for 700 nm to 1100 nm.

Thanks to non-linear optics, it is possible to convert the wavelength of solid state lasers into the visible and the ultraviolet. In fact, when the electric field intensity is very high, as is the case for laser waves, matter does not respond linearly to the electromagnetic excitation of light. It responds by emitting new frequencies. Figure 23 shows that it is possible to generate new frequencies in a water cell if the laser is intense enough.

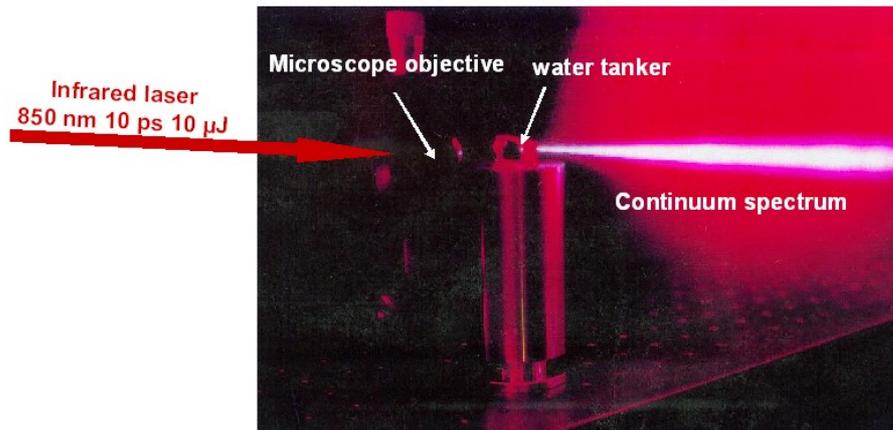


Figure 23: Non-linear effect (frequency continuum) with a picosecond pulsed laser focused in water with a diameter of a few microns (the energy is  $10 \mu\text{J}$ ).

Figure 24 illustrates another example of the non-linear effect created in a standard optic fibre when the peak power density exceeds  $\text{GW}/\text{cm}^2$ : a green beam (532 nm) is injected into the fibre. New frequencies are generated in the orange and in the red by the Raman effect.

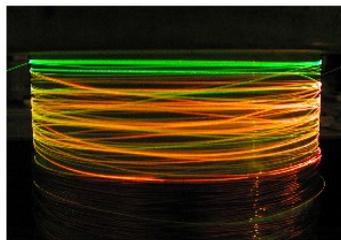


Photo : F.Druon et S.Forget Institut d'Optique

Figure 24: Non-linear effect in an optic fibre.

These non-linear effects vary according to the nature of the materials. To promote this effect, so-called non-linear crystals are used. Figure 25 shows another example of generating frequencies in the visible, this time from a non-linear crystal. The most commonly used non-linear effect is frequency doubling, particularly for the conversion of  $1064 \text{ nm}$  (the Nd : YAG laser)  $\rightarrow 532 \text{ nm}$  (emits in the green).



Photo : S.Equibey L. Mc Donagh et F.Balembois Institut d'Optique

Figure 25: Generation of visible frequencies in a non-linear crystal (optical parametric oscillator).

Solid state lasers differ in the geometry of their amplifying media: some are large (generally crystals) of millimetric dimensions and there are optic fibres that can be several metres long. The diode pumped solid state lasers, and particularly the fibre lasers, are extremely robust and have a lifetime longer than 10,000 hours. They are highly valued for their industrial applications (welding, marking). Their compactness is an added advantage.

## F. Some examples of applications

Lasers are now found in many sectors (optical telecommunications, storage of information (CD, DVD), instrumentation, measurement, biomedical, material processing). They are becoming increasingly reliable, compact and powerful. The range of applications will therefore expand. There are already so many that it is quite difficult to give an exhaustive list. However, they can be classified according to the properties of the laser radiation. As stated previously, it is a spatial, spectral and temporal concentration of light.

### 1. Spatial concentration

The laser beam can have a very low divergence. This characteristic can be used to project light very far from its source. In this way, thanks to the laser, it is possible to send light to the moon and to collect some photons reflected back by reflectors placed on the moon's surface during the Apollo missions. By measuring the time it takes for light to go to the moon and back, the laser enables the precise calculation of the distance from the earth to the moon.

Due to its low divergence, the laser is used in telemetry to measure at long range (up to several hundreds of kilometres) the concentration or nature of gases, wind speed, or the distance to an obstacle or a target. The associated instrument is thus called a LiDAR (for Light Detection and Ranging) similar to radar in the field of radio waves.

The laser can also be focused at dimensions of the order of a wavelength. This characteristic is used for either treating materials (drilling, cutting, marking) or analysing them (for example, confocal microscopy).



**Photo : F. Druon Institut d'Optique**

*Figure 26: An example of the machining of a metal plate by a picosecond laser: the light is a plasma caused by the ejection of matter. The metal plate to be machined is on the left.*

## **2. Spectral concentration**

The laser is capable of emitting radiation at an extremely precise frequency: the relative precision  $\Delta\nu/\nu$  can reach  $10^{14}$ . It is therefore a reference of frequency that can now be quite easily linked to the frequency standard of the caesium clock. Thanks to lasers, spectroscopy has made enormous progress. The spectral fineness of laser sources now enables atoms to be manipulated and to be cooled down. Thus, knowledge in optics can be transposed to the study of atoms (atomic optics).

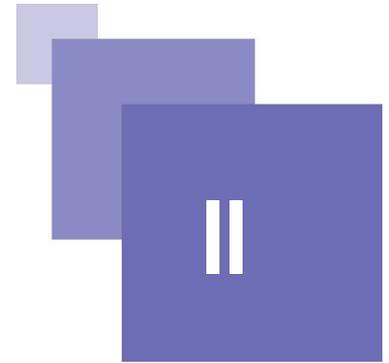
## **3. Temporal concentration**

The shortest events ever created by man are obtained by lasers interacting with gases. Currently, these time periods are less than a femtosecond. Phase-synchronised (mode-locked) lasers usually produce picosecond or femtosecond (commercial lasers) pulses, which are markedly shorter than the events to be analysed. They can thus be used to sample temporally a very fast phenomenon, such as a chemical reaction involving the formation of free radicals.

Q-switched lasers (nanosecond pulses) can be used to immobilise particles in a gas or a moving machine part (the stroboscope effect). The temporal concentration enables instantaneous electrical fields to be achieved, which can be greater than the fields of atomic bonds in a molecule or a metal. It is therefore possible to create plasmas or to detach atoms from a surface without producing any warming.



# Case study: Diode-pumped Nd: YAG Laser



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The aim of this case study is to look at how to build, in practical terms, a diode-pumped Nd:YAG laser emitting at 1064 nm. Some concepts will be explained in more depth with a view to putting them into practice. The orders of magnitude and key technological points will be presented.

## A. Spectroscopy of the neodymium ion

### 1. Presentation of the energy levels

The neodymium ion ( $\text{Nd}^{3+}$ ) in the YAG matrix ( $\text{Y}_3\text{Al}_5\text{O}_{12}$ , yttrium aluminium garnet) has a great many levels that can give different laser transitions. Figure E1 locates the energy levels by wavenumber  $\sigma$  expressed by convention in  $\text{cm}^{-1}$ . The wavelength  $\lambda$  (in m) corresponding to a transition between levels 1 and 2 is obtained from:  $\lambda = 10^{-2} / (\sigma_2 - \sigma_1)$ .

The energy levels of the neodymium ion  $\text{Nd}^{3+}$  are represented by a group of letters and numbers that give the quantum numbers associated with different components: the letter corresponds to the orbital quantum number, the superscript number gives the spin quantum number and the subscript fraction is the angular quantum number. Because of the crystal field (Stark effect), the energy levels are split into sublevels, represented by letters with subscripts ( $Z_1 \dots R_2$ ).

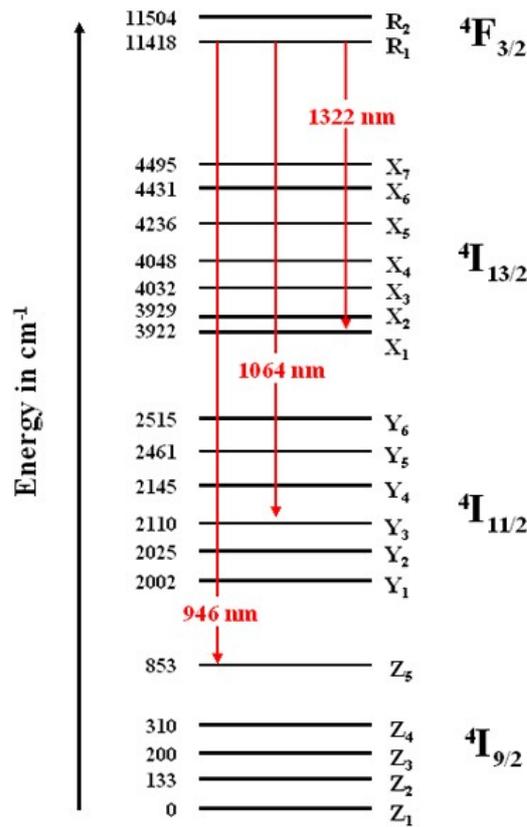


Figure E1: Different levels of the neodymium ion. (There are also higher energy levels, not shown here as they are not involved in laser emission)

The neodymium ions only stay a long time in the level  $4F_{3/2}$ . The lifetime of this level is around  $230 \mu s$  while it is less than a nanosecond in the other levels. Thus, the ions accumulate in this level and can fall from it by intense stimulated emission.



### Définition

The lifetime of an atom in a level is the average time it is present in this level before de-excitation. It can be shown that, if the population of the level is  $N_0$  at  $t=0$  s, then

$$N(t) = N_0 \exp(-t/\tau), \text{ at the instant } t, \text{ where } \tau \text{ is the lifetime.}$$

Figure E<sub>1</sub> shows a great many levels and thus numerous possibilities for laser emissions and transitions from level  $4F_{3/2}$ . The red arrows give the wavelengths of the most used laser transitions: 1064 nm corresponds to the transition with the greatest probability of stimulated emission. There is also a line in the further infrared towards 1320 nm. Finally, the Nd:YAG has another quite efficient transition in the near infrared at 946 nm.

## 2. Population of "lower levels" at room temperature

In order to find out the efficiency of a laser transition, it is important to know if the lower level is occupied or not at thermodynamic equilibrium. For this, Boltzmann's Law is applied:

$$N_1 = N_0 \times \exp(-E_1/kT)$$

where  $N_0$  is the population of the ground state whose energy is taken to be 0 and  $N_1$  is the population of

the energy level  $E_1$  under consideration. To make this easier to use, “thermal” energy  $kT$  can be converted into  $\text{cm}^{-1}$  thanks to the formula given in the note below. Thus, for a temperature of 300 K,  $kT=208 \text{ cm}^{-1}$ .



### Remarque

From the relation between frequency and energy level:  $E = h\nu$ , the relation between energy expressed in wavenumber and energy expressed in joule can be deduced:

$$E(\text{J}) = 100 h c E(\text{cm}^{-1})$$

By applying Boltzmann's Law, it can thus be shown that the lower levels for the transitions at 1064 nm and 1320 nm are not occupied because they are situated several thousands of  $\text{cm}^{-1}$  from the ground state: the ratio  $E_1/kT$  is thus very low

On the other hand, the lower level ( $Z_s$ ) for the transition at 946 nm has an energy of the same order of magnitude as  $kT$ . 1.6% of the population of the ground state is found in the lower level. Thus, to invert the population, at least the same quantity of ions must be put into the upper level and this quantity cannot then be used for amplification by stimulated emission. This results in a loss of efficiency in comparison with the previous transitions.

### 3. The system operating at 1064 nm pumped by a diode at 808 nm

The neodymium ion also has other levels at a higher energy than the level  ${}^4F_{3/2}$ . (these are not shown in Figure E1 for reasons of simplicity). For example, the level  ${}^4F_{5/2}$  allows the absorption of light at 808 nm. From level  ${}^4F_{5/2}$ , the ions drop to level  ${}^4F_{3/2}$  without radiating. So, the pumping transition ( ${}^4I_{9/2}$  to  ${}^4F_{5/2}$ ) occurs over two different levels from those of the laser transition ( ${}^4F_{3/2}$  to  ${}^4I_{11/2}$ ). The system is thus at four levels (Figure E2).

It should also be noted that the ions do not accumulate in the lower level once they have lost their energy in the form of light: the passage between level  ${}^4I_{11/2}$  and the ground state is very fast.

The cycle of a neodymium ion is summarised in Figure E2. It is, in fact, an ideal spectroscopic diagram as all the excited atoms accumulate in the upper level while the lower level is never occupied, neither at thermodynamic equilibrium nor during the laser operation.

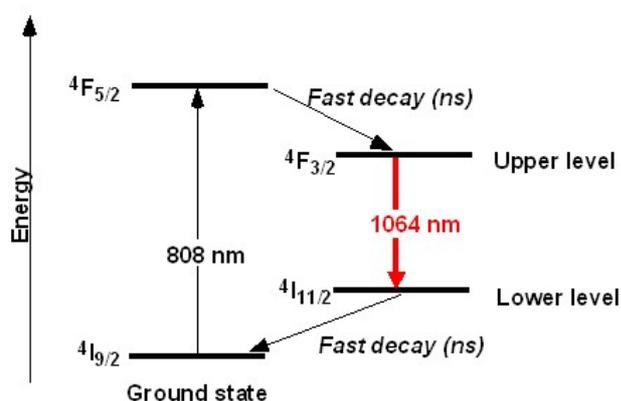


Figure E2: Simplified diagram for the transition at 1064 nm.

## B. The amplifying medium pumped by a diode

### 1. Experimental setup

Pumping at 808 nm is carried out by a diode laser, which emits a power of 500 mW over a rectangular emitting surface (  $1 \mu m$  by  $100 \mu m$  ). The pumping radiation is collected by a lens, which returns the image of the emitting surface to infinity (collimation). It is then focused in the Nd:YAG crystal.

The magnification of the optical system (collimation and focusing) is equal to 1. The beam emitted is very divergent ( $50^\circ$ ), so the objective lens used has to be very wide to collect all the flux emitted by the source. This is why a collimating lens with a numerical aperture of 0.5 is used here.

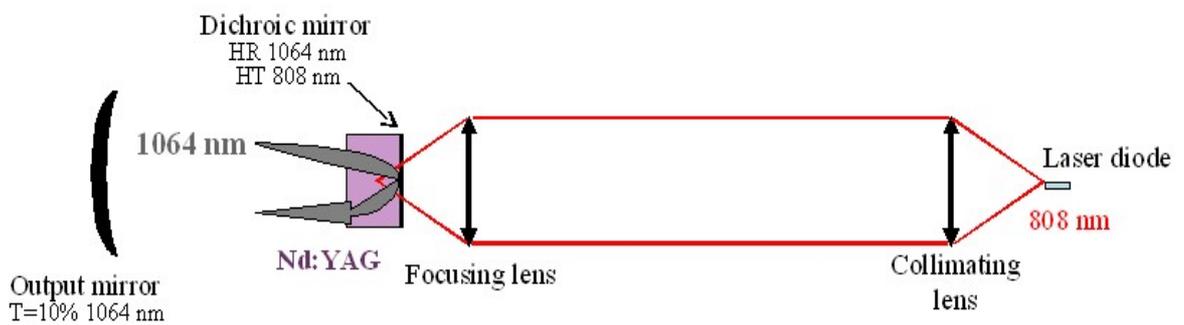


Figure E3: Diagram of the pumping optics

The crystal is 10 mm in length. The optical axis of the cavity continues along the optical axis of the pumping and the process is known as longitudinal pumping.

The crystal has a dielectric coating on its entry side. This consists of a mirror coating at wavelength 1064 nm and an anti-reflection at 808 nm so that the beam from the pump travels through the crystal while the beam from the laser is reflected.

The focal point in the laser crystal is about  $20 \mu m$  by  $100 \mu m$  (optical aberrations mean that the narrowest rectangular section of the diode (  $1 \mu m$  ) is not imaged correctly ). The focal point may seem very small, but the pump beam must be focused in the crystal for the pumping to be efficient and for the effective gain to be high enough so that laser oscillation can occur. In fact, when the intensity of the laser beam is small, the effective gain  $G_0$  is linked to the irradiance of the pump  $I_p$  on the crystal by the following formula (following the hypothesis that the pump beam and the laser beam have the same section):

$$G_0 = \exp(C_{ste} \cdot I_p)$$

where  $C_{ste}$  is a constant defined by the spectroscopic parameters of the crystal and the size of the beams. The gain in the system shown in Figure E3 can easily be measured experimentally. For a 500 mW pump focused onto a surface about hundred microns wide,  $G_0$  is roughly 1.5 at 1064 nm for the Nd:YAG crystal.

## C. Placing the Nd:YAG crystal in the cavity

To build a laser oscillator, an output mirror needs to be placed facing the mirror coated on the Nd:YAG. The choice of this mirror is important in terms of its transmission at 1064 nm, its reflectivity at other wavelengths and its radius of curvature.

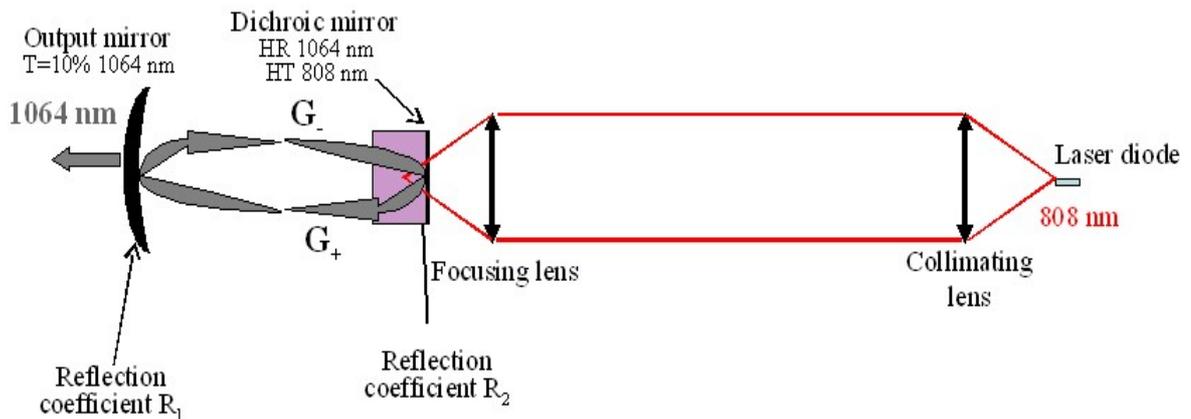


Figure E4: Diagram illustrating the complete set-up with the output mirror.

### 1. Power of the pump at the lasing threshold

#### a) Output mirror transmission

The output mirror transmission should be chosen according to the gain available in the amplifying medium. As stated in the lesson, the product of the gains in both directions  $G_+G_-$  must be more than  $1/R_1R_2$  (see Figure E4 for the sizes) so that laser oscillation can occur. Here, the mirror coated on the Nd:YAG crystal is assumed to be very reflective so  $R_2=1$ . However, energy loss can occur in the cavity at the crystal-air interface or by diffusion via dust on the mirrors. By convention, these losses are taken into account by giving the second mirror  $M_2$  a reflection coefficient slightly less than 100%. Generally, these so-called passive losses make up 1 to 2% in this type of laser cavity so  $R_2$  is about 98%. As the output mirror transmission of  $M_1$  is very small ( $T_1=10\%$ ), the intensity of the laser will not change much before and after the crystal so it can be assumed that  $G_+=G_-$ .

As  $R_1=1-T_1$ , the condition for oscillation is:  $G_2>1/R_1R_2$ .

When the pump is at full power and for a signal at 1064 nm,  $G_0^2$  is equal to 2.25 according to the order of magnitude given in the section “The amplifying medium pumped by a diode”. The fraction  $1/R_1R_2$  is equal to 1.13 thus the the lasing threshold is exceeded.

#### b) Power of the pump at the lasing threshold

It is possible to calculate the power of the pump  $P_p$  necessary to reach the oscillation or lasing threshold ( $G_0^2=1/R_1R_2$ ).  $G_0$  must be given as a function of the power of the pump by using the formula from the section “The amplifying medium pumped by a diode”:

$$G_0 = \exp(P_p \ln G_{0max} / P_{Pmax})$$

where

$$G_{0max} = 1.5 \quad \text{and} \quad P_{Pmax} = 500 \text{ mW}$$

Thus

$$P_{Pthresh} = 1/2 (P_{Pmax} / \ln G_{0max} \ln(1/R_1 R_2))$$

so the pump power at the threshold is 77 mW.

### c) Why are the laser beams so small ?

The beams at 808 nm and 1064 nm have a radius of about  $70 \mu m$  inside the crystal. This may seem very small but it is necessary so that the number of ions per unit of volume is sufficient and also so that the number of photons at 1064 nm is enough to trigger an efficient stimulated emission. The formula that calculates the gain according to the irradiance can be modified to include the power of the pump and the radius of the pump beam,  $r: I_p = P_p / (\pi r^2)$ . Assuming that the beams at 808 and 1064 nm have the same radius, it is possible to calculate the minimum radius needed to reach the oscillation threshold with an output mirror transmitting 10% and a maximum pump power.

Using the formula  $G_0 = \exp(Cste \cdot I_p)$  with the following conditions given in the section "The amplifying medium pumped by a diode":  $G_{0max} = 1.5$  for a pump power of  $P_{Pmax} = 500 \text{ mW}$  focused in the crystal with a radius of  $r_{max} = 70 \mu m$ ,

then

$$Cste = \ln(G_{0max}) \pi r_{max}^2 / P_{Pmax}$$

To be at the threshold with the maximum pump power,  $G_0^2$  must be equal to  $1/(1-T)$

where

$$G_0 = \exp(\ln G_{0max} r_{max}^2 / r^2)$$

then the value of the radius  $r$  is calculated as  $178 \mu m$ . This means that if the diameter of the beams is more than this value, then the power of the pump is not high enough to reach the oscillation threshold. The beam radius must therefore be much less than a millimetre.

## 2. Selection of the laser transition at 1064 nm, choice of dielectric mirrors.

The section "Spectroscopy of the neodymium ion" demonstrated that the Nd:YAG can function over several laser transitions, specifically three bands: 1064 nm, 946 nm and 1320 nm. The transition corresponding to 1064 nm has by far the highest effective gain so the laser naturally tends to operate at 1064 nm.

However, to avoid unwanted oscillations, it is better to monitor the reflection coefficients of the mirrors so that  $1/R_1(\lambda)R_2(\lambda)$  is greater than  $G_0^2$  at the undesirable wavelength,  $\lambda$ . This means the lasing threshold will not be reached.

The dielectric treatment used for the mirrors is based on the principle of interference: it consists of a deposit of thin layers (for example, alternating layers of  $\text{SiO}_2$  and  $\text{TiO}_2$ ) so that certain wavelengths

interfere constructively and others do not. Dielectric mirrors have a band of reflectivity that usually stretches over tens of nanometres in wavelength. On either side of this band, the mirror is transparent. Figure E5 shows the classic reflectivity curve for a mirror reflecting at 1064 nm and a photo of such a mirror placed on a white sheet. The sheet is clearly visible through the mirror, proof that the mirror is transparent in the visible spectrum while it is completely reflecting in the near infrared.

### Reflectivity curve of a standard mirror

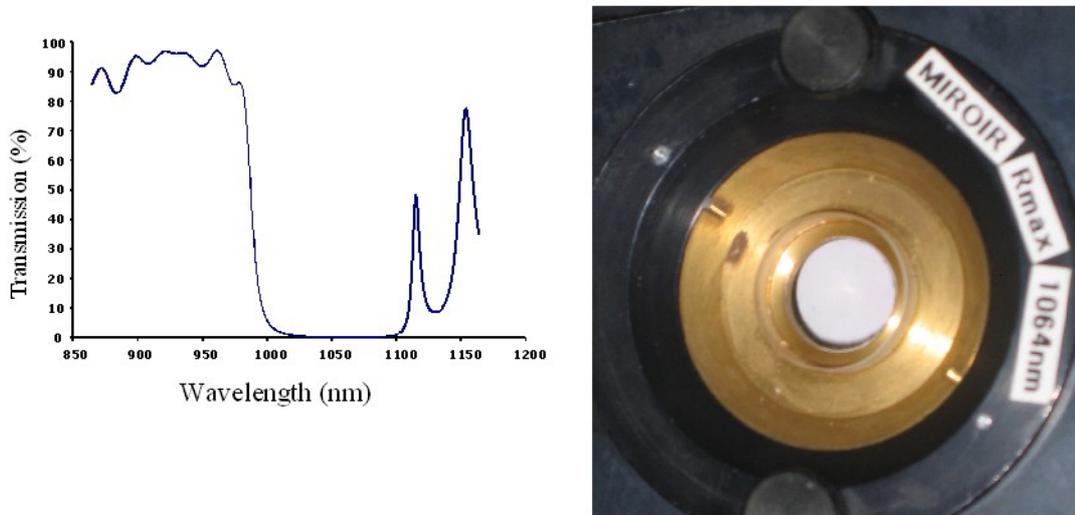


Figure E5: Photo of a highly reflective mirror at 1064 nm and its associated reflectivity curve.



### Remarque

The mirrors used in Nd lasers are never metallic as these have a reflection coefficient of 97% (the rest is absorbed by the metal). Thus, not only do they create unwanted losses but they can also become hot in the laser and may even change beam shape slightly.

### 3. Choice of the radius of curvature of the mirror and the laser mode in the cavity

The cavity shown in Figure E4 is “planar-concave” composed of a planar mirror placed on a Nd:YAG crystal and a concave output mirror. The radius of curvature of the mirror and the distance between the two mirrors are specifically chosen so that a Gaussian wave can propagate indefinitely in the cavity while keeping the same shape throughout the cavity.

Figure E6 shows the form of this wavefront at several points in the cavity. Its radius of curvature closely follows the shape of the end mirrors: concave on one side and flat on the other. For such a Gaussian wave to exist in a planar-concave cavity, the length of the cavity must be shorter than the radius of curvature of the concave mirror. Then the cavity is said to be stable.

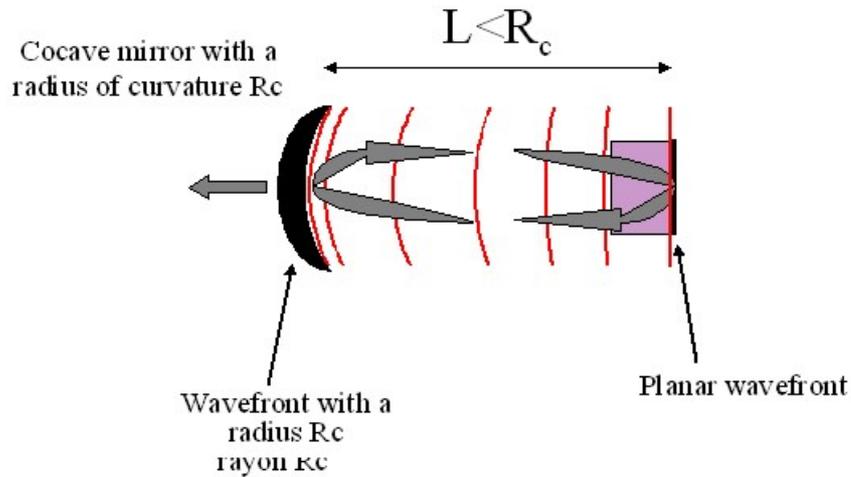


Figure E6: Form of the Gaussian wavefront propagating indefinitely in the cavity.

## D. Setting up the laser

### 1. The pumping laser diode

This section describes how a laser is built and the different components that are used.

The box of the diode laser is illustrated in Figure E7. The insert shows the diode itself and the rectangular  $1\ \mu\text{m}$  by  $100\ \mu\text{m}$  emitting surface, which is placed on a vertical stand inside the box. The box also contains a Peltier element to regulate the temperature and it is mounted on a metallic radiator to evacuate the heat that is created by the operation of the diode. An electrical current of 1 A at 2 V needs to be injected into the system so that the diode will emit 500 mW at 808 nm.

Figure E7 also shows all the electric wires needed to control the current injected into the diode and its temperature.

Temperature control is important because the wavelength of the laser varies by about 0.3 nm per  $^{\circ}\text{C}$ . The heating of the junction of the diode depends on the current injected so the spectrum can easily vary by more than 1 nm. This has to be taken into account because the absorption spectrum of Nd:YAG is centred around 808 nm with a width of the order of nm so a slight change in wavelength will result in a decrease in absorption (the pump photons will no longer correspond to the pump transition) and consequently a decrease in the effective gain.

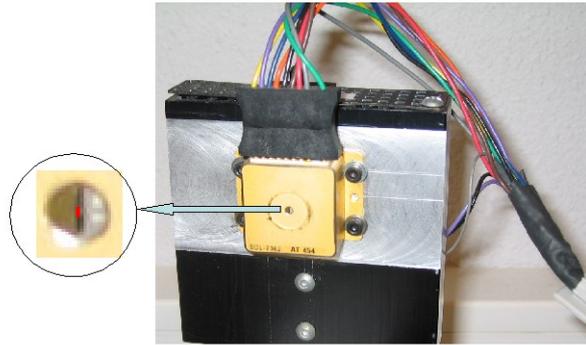


Figure E7: Photo showing the box containing the diode laser.

## 2. The Nd :YAG laser

The whole laser is shown in Figure E8: the pumping system optics and the cavity. The photo also shows that the different components are positioned on adjustable mounts: the collimation and focal lenses can be moved along three different axes.

It is important to understand that these movements must be very precise. In fact, the beams in the Nd:YAG crystal have a radius of only  $70\ \mu m$ . For the laser beam and the pump beam to be in the same place, the latter must be capable of being moved in the perpendicular plane of the optical axis with a precision of tens of microns.

The mirrors of the cavity (Nd:YAG on one side and the output mirror on the other) are mounted in supports that can be tilted. The mirrors must be aligned with great precision so that the Gaussian wave can travel back and forth indefinitely in the cavity. For the correct order of magnitude of the angular precision of the output mirror, its optical axis must cross the pumping zone in the crystal. This zone is only about a hundred microns so an adjustment to the nearest  $10\ \mu m$  is needed when the output mirror is about 7 cm from the Nd:YAG. This gives an angle of 0.1 mrad

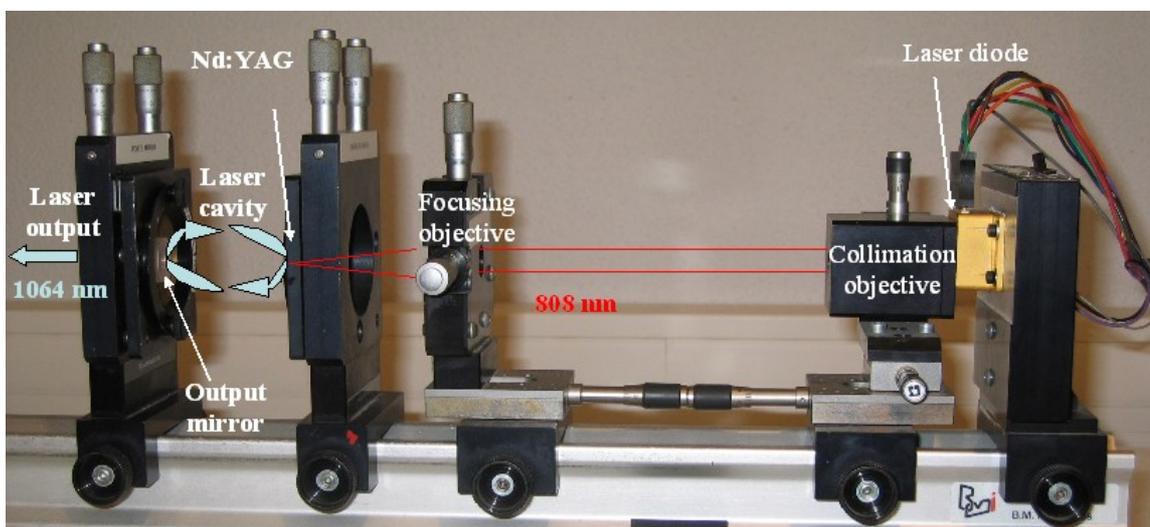


Figure E8: Photo showing all the components needed to build a laser (built by BM Industries) .

In practice, the pump beam is first collimated then aligned parallel to the bench. This defines a reference axis (the optical axis), which will be used to align the optical cavity mirrors. This is known as “co-linear” pumping where the pumping axis and the optical axis are the same.

Thus, this beam is used to adjust the two mirrors by autocollimation. Next, the pump beam is focused in the Nd:YAG crystal. By placing the output mirror at a distance shorter than the critical distance ( $R_c$ , here

## Case study: Diode-pumped Nd: YAG Laser

equal to 100 mm), the laser effect can be achieved.

This effect is characterised by an intense light invisible to the naked eye (at 1064 nm) but detectable by a CDD camera or a simple camera as shown in Figure E9. This photo is taken from outside the optical axis. The laser photons that hit the detector are those diffused by the mirror. There are not many of them compared to the photons aligned along the optical axis. However, there are enough to create a signal on the detector of the same order of magnitude as that of the mounts and the surrounding room. In fact, if the camera had been placed in the path of the output beam, it would have been dazzled and probably damaged. Figure E9 also shows that the laser beam has a weak spatial extension compared to the size of the mirror: a radius of about 1 mm.

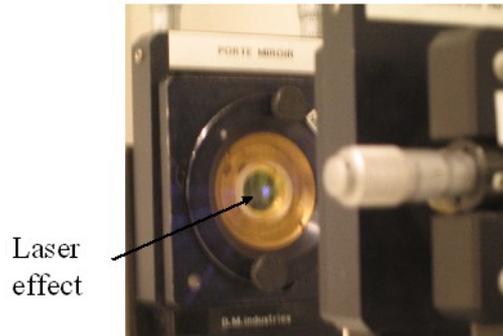


Figure E9: Photo of the output mirror during the laser effect. The focal point is invisible to the naked eye but can be detected by a digital camera

## E. Output power

### 1. Gain

Before calculating the output power, it is interesting to see how the effective gain varies according to the power of the pumping system. Below the oscillation threshold, the effective gain varies exponentially as a function of  $P_p$ .

$$G = \exp(C_{ste} P_p / \pi r^2)$$

At the threshold and above it when the laser oscillates, the effective gain follows  $G^2 = 1/R_1 R_2$ . It is thus fixed at a value determined by the transmission coefficients of the mirrors and the passive losses of the cavity. Figure E10 shows the curve of this evolution as a function of the pump power. When the pump power is zero, the effective gain is 1. This is because the lower energy level of the laser transition is empty so there can be no absorption. Figure E10 also shows the evolution of the effective gain if there is no optical cavity: it rises exponentially.

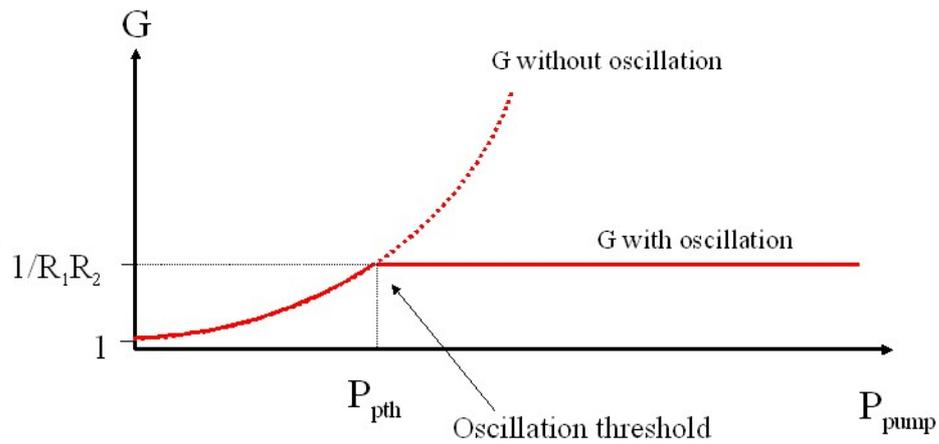


Figure E10: Form of the effective gain  $G$  in the Nd:YAG crystal as a function of the pump power

## 2. Output power formula

Beyond the oscillation threshold, the power at 1064 nm is no longer negligible. To simplify the process, each pump photon beyond the oscillation threshold is assumed to become a laser photon and leave the cavity. To do this, the photons must either go through the output mirror or experience passive losses in the cavity. Figure E11 shows where the different laser outputs are situated. As seen in the photo of Figure E9, there are losses by diffusion on the mirrors and globally over all the interfaces. These losses, as well as the residue of the transmission through the mirror  $M_2$ , cannot be used: they are the so-called passive losses. The only useful part of the beam is that which leaves via  $M_1$ . The intensity of this beam is called the output power or  $P_{out}$ .

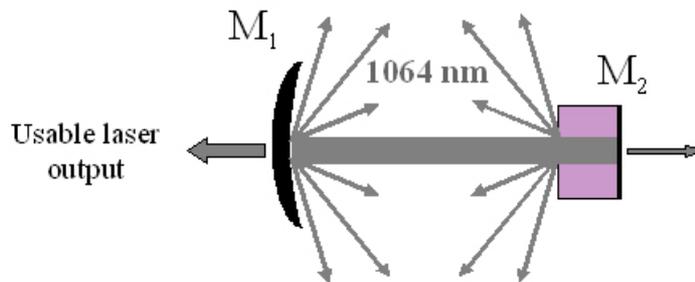


Figure E11: Laser cavity outputs

As explained in the section “Power of the pump at the lasing threshold”, all the passive energy losses are grouped together in the value for the transmission of the output mirror  $M_2$ . This value  $T_1$  is given by:  $T_1=1-R_1$  and  $T_2$  is defined by  $T_2=1-R_2$ . If  $P_{intern}$  is the power circulating inside the cavity, then the total power emitted at 1064 nm,  $P_{emitted}$ , is given by:

$$P_{emitted} = T_1 P_{intern} + T_2 P_{intern}$$

The output power,

$$P_{out} = T_1 P_{intern}$$

**Case study: Diode-pumped Nd: YAG Laser**

so compared to the total emitted power,

$$P_{out} = P_{emitted} T_1 / (T_1 + T_2)$$

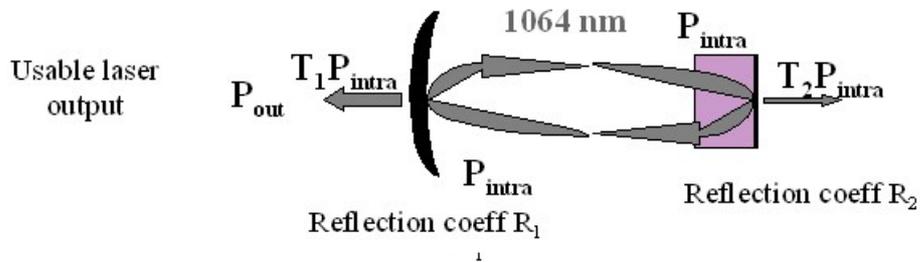


Figure E12: Different abbreviations and units used to calculate the output power

Above the laser threshold, all the pump photons are assumed to be absorbed by the Nd:YAG crystal and then transformed by stimulated emission into laser photons. The number of pump photons converted per second is equal to:

$$( P_P - P_{P_{thresh}} ) / h \nu_P$$

If the number of laser photons emitted is equal to

$$P_{emitted} / h \nu_{then} ( P_P - P_{P_{thresh}} ) / h \nu_P = P_{emitted} / h \nu$$

Finally, the output power can be expressed thus:

$$P_{out} = (T_1 / T_1 + T_2) (\lambda_p / \lambda) (P_P - P_{P_{thresh}})$$

**3. Value for the output power**

Figure E13 shows the efficiency curve of the laser: the output power as a function of the pump power. According to the preceding formula, the curve is actually linear with a slope that depends on two parameters:

- The part of the beam passing through the output mirror compared to the total losses of the cavity: to maximise the output power, these losses must be minimised.
- The ratio of the pump wavelength to the laser wavelength. This value is specific to each laser system and for this one is equal to 0.76

The slope of this line is often called the laser efficiency and in this case is equal to 63%. By using the pump power at the lasing threshold calculated previously (see the section “Power of the pump at the lasing threshold”), the output power can be calculated as 266 mW (if the maximum pump power is equal to 500 mW). This value is fairly close to what is found experimentally.

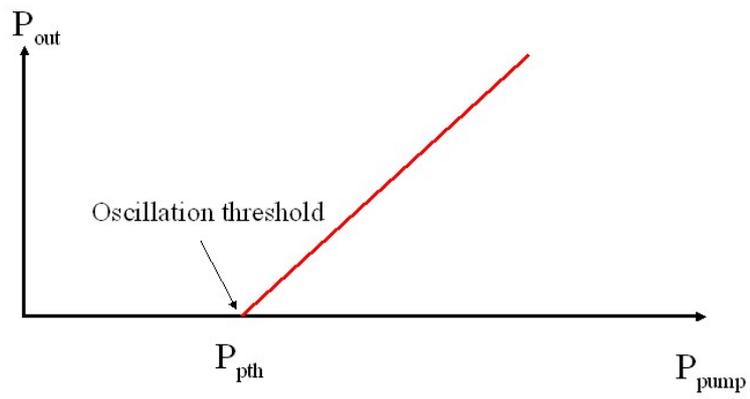
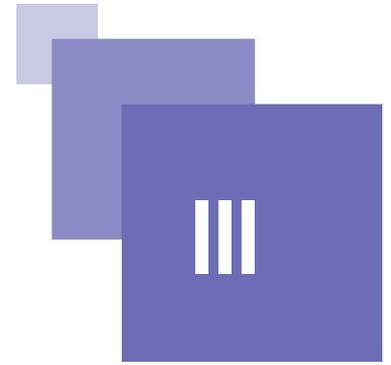


Figure E13: Efficiency curve of the laser: the output power as a function of the pump power



# Lesson questions



## A. Problem

Please answer the following questions:

### Question 1

A helium-neon laser emitting at 633 nm makes a spot with a radius equal to 100 mm at  $1/e^2$  at a distance of 500 m from the laser. What is the radius of the beam at the waist (considering the waist and the laser are in the same plane)?

### Question 2

A ring laser cavity is composed of 4 identical mirrors with a reflection coefficient equal to 0.99. Determine the value of the gain needed in the amplifying medium to obtain laser oscillation.

### Question 3

Calculate the gap in frequency between two longitudinal modes in a linear cavity whose optic length,  $L$ , =300 mm.

### Question 4

What is the rate of repetition of the pulses emitted by a mode-locked laser? The optic length of the cavity,  $L$ , is 1 m.

### Question 5

A mode-locked laser emits an average power  $P$  equal to 1 W. The rate of repetition of the pulses from this laser is equal to 100 MHz. Calculate the energy of each pulse.

### Question 6

Consider a lower energy level situated  $200 \text{ cm}^{-1}$  from the ground state. There are no other energy levels nearby. Determine the fraction of the population found in this level compared to the ground state population at a temperature of 300 K.

Boltzmann's constant is equal to  $1.38 \cdot 10^{-23} \text{ JK}^{-1}$

The conversion from  $\text{cm}^{-1}$  to joules is given by:  $E(J) = 100 h c E(\text{cm}^{-1})$ , where  $h$  is Planck's constant ( $6.62 \cdot 10^{-34} \text{ Js}$ ) and  $c$  is the speed of light in a vacuum ( $3 \cdot 10^8 \text{ ms}^{-1}$ )

### Question 7

Consider an optical pump at 940 nm for a Yb:YAG crystal placed in a laser cavity. The wavelength of ytterbium is 1030 nm. If all the photons emitted by the pump are absorbed by the crystal and used for the

## Lesson questions

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lasing process, calculate the maximum power output. The pump power is 1 W.

### Question 8

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The amplifying medium of a helium-neon laser has an amplification spectral band equal to  $\Delta\nu = 1 \text{ GHz}$  at 633 nm. For simplicity, the spectral profile is assumed to be rectangular. The linear cavity is 30 cm long. Calculate the number of longitudinal modes that can oscillate in this cavity.

### Question 9

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CO<sub>2</sub> has a bandwidth equal to  $\Delta\nu = 50 \text{ MHz}$  at  $10.6 \mu\text{m}$ . For simplicity, the spectral profile is assumed to be rectangular. The length of the cavity is equal to 1 m.

1. Calculate the number of longitudinal modes that can oscillate in this cavity.
2. At what distance must the cavity mirror be placed so that at least one of the modes falls in the amplification band?

### Question 10

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A Q-switched laser emits pulses of  $10\mu\text{J}$  of duration 1 ns. The repetition rate of the pulses is equal to 10 kHz.

1. Calculate the peak power of the pulses.
2. Calculate the average output power of the laser.